

Note: In this exam there are special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points — 1st Exam

Instructions

א. Duration of the exam: Three and a half hours.

ב. Exam structure and breakdown of points:

This exam has three parts with a total of eight questions.

Part One: Algebra, Analytical Geometry, Probability

Part Two: Planar Geometry and Planar Trigonometry

Part Three: Differential and Integral Calculus of
Polynomials, Rational Functions, and Root
Functions

Answer **five** questions of your choice — $5 \times 20 = 100$ points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the
programming options of a programmable calculator.
Use of a graphing calculator or programming options
may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number
of the question you are answering.

(2) Start each question on a new page. Write the steps of
the solution in the answer booklet, even when the
calculations are done with a calculator.

Explain all of your work, including calculations, in
detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the
disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each
draft page.

If you write draft material on paper outside the answer booklet,
your exam may be disqualified.

מתמטיקה

4 יחידות לימוד-שאלון ראשון

הוראות

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון — אלגברה, גאומטריה אנליטית, הסתברות

פרק שני — גאומטריה וטריגונומטריה במישור

פרק שלישי — חשבון דיפרנציאלי ואינטגרלי של פולינומים,
של פונקציות רציונליות ושל פונקציות שורש

יש לענות על **חמש** שאלות לבחירתכם — $5 \times 20 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון
שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול
לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. יש לרשום במחברת את

שלבי הפתרון, גם כאשר החישובים מתבצעים
בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט
ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת
הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד
המשמש טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת
הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer five of the questions 1-8 (each question — 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry, Probability

1. The distance between City A and City B is 75 km.

A motorcyclist set off from City A toward City B.

At the same time a cyclist set off from City B toward City A.

The motorcyclist and the cyclist both traveled along the same route.

The motorcyclist traveled at a constant speed of 80 km/h. The cyclist traveled at a constant speed of 20 km/h.

- א. How much time after they set off did the motorcyclist and the cyclist meet?

The motorcyclist reached City B, then headed straight back toward City A.

On his way back to City A, the motorcyclist met the cyclist for the second time.

Both of them were traveling at the same speed as they had traveled at previously.

- ב. How much time passed between the **first** meeting and the **second** meeting of the motorcyclist and the cyclist?
- ג. What distance did the cyclist travel from the moment he set off until he met the motorcyclist for the **second** time?

2. The figure on the right shows a circle whose equation is $(x - 8)^2 + y^2 = 25$.

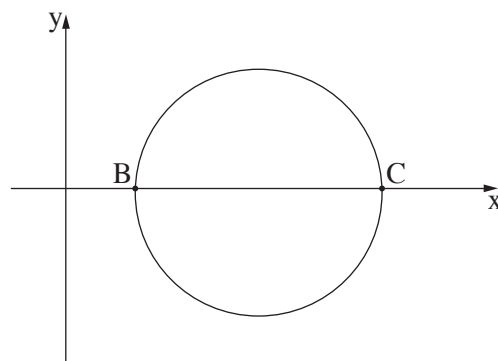
Points B and C are located on the x-axis, as shown in the figure.

- א. Find the coordinates of Points B and C.

Point A is located on the circle in the fourth quadrant.

Given : The area of Triangle ABC is 20.

- ב. (1) Find the length of the height to Side BC in Triangle ABC.
- (2) Find the coordinates of Points A (two possibilities).



Given : The slope of the tangent to the circle through Point A is positive.

- ג. Find the equation of the line that is tangent to the circle through Point A.
- ד. Find the area of the quadrilateral bounded by the tangents to the circle through Points A, B and C, and the x-axis.

/Continued on page 3/

3. A company has selected candidates for a job in the company.

In order to be hired by the company, candidates have to pass the three stages of the selection process.

Candidates who did not pass the first stage, did not move on to the second stage, and candidates who did not pass the second stage did not move on to the third stage.

The stages of the selection process were as follows :

First stage: Screening test

Second stage: Personal interview

Third stage: Group workshop

All the candidates who passed **all** three stages were hired to work for the company.

Given:

75% of the candidates passed the screening test.

50% of the candidates who passed the screening test, passed the personal interview.

40% of the candidates who passed the personal interview, passed the group workshop.

- א. One candidate was randomly chosen. What is the probability that he was hired to work for the company?
- ב. Neta and Gali were candidates who went through the selection process. What is the probability that no more than one of them was **hired** to work for the company?
- ג. Adi was also a candidate in the selection process. What is the probability that she passed the personal interview, if we know that she was **not** hired to work for the company?
- ד. We know that 170 candidates of all the candidates were not hired to work for the company. How many candidates were hired to work for the company?

Part Two — Planar Geometry and Planar Trigonometry

4. BA is a diameter of a circle whose center is O (see figure).

Points C and D are located on the circle such that

$$\angle BOC = 2 \cdot \angle AOD \text{ is satisfied.}$$

- א. Prove that $\angle CAB = \angle AOD$.

Point E is located on the extension of the diameter BA, as shown in the figure.

Given: ED is a tangent to the circle through Point D.

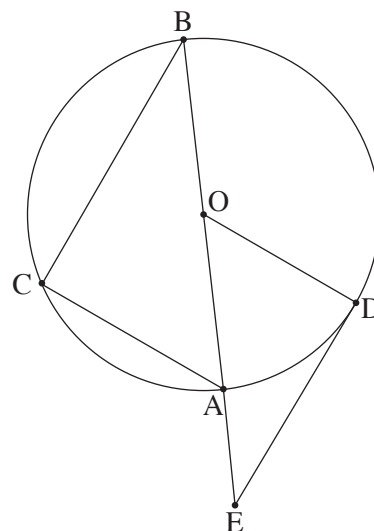
- ב. Prove that $CB \parallel ED$.

- ג. Prove that $BA \cdot OD = OE \cdot AC$.

We know that the area of Triangle CAB is 1.44 times greater than the area of Triangle DOE.

Let R denote the radius of the circle.

- ד. Express the length of Segment AE in terms of R.

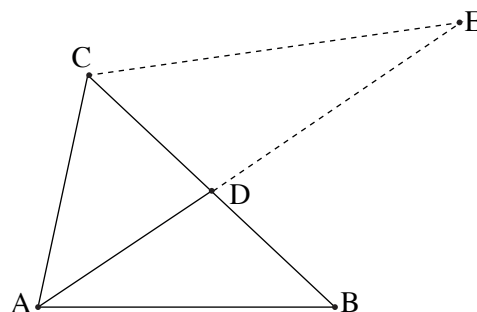


5. The figure below shows Triangle ABC, in which the length of the sides are :

$$AC = 6, AB = 7, BC = 8.$$

AD is the median of Side BC in Triangle ABC.

- א. (1) Find the size of the angle $\angle ABC$.
(2) Find the length of the median AD.
(3) Find the size of the angle $\angle BAD$.



Point E is located on the extension of AD, as shown in the figure.

Given: The area of Triangle CDE is 16.

- ב. Find the length of DE.

A perpendicular to the median AD is drawn through Point C, intersecting AD at Point F.

- ג. Find the ratio between the area of Triangle CDF and the area of Triangle CDE.

Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the given function $f(x) = \frac{4x^2 - 1}{x^2 - 1}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
- (3) Find the coordinates of the intersection points [נקודות החיתוך] of the graph of function $f(x)$ with the axes.
- ב. Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
- ג. Sketch a graph of the function $f(x)$.

Consider the given function $g(x) = -f(x) + k$, k is a parameter.

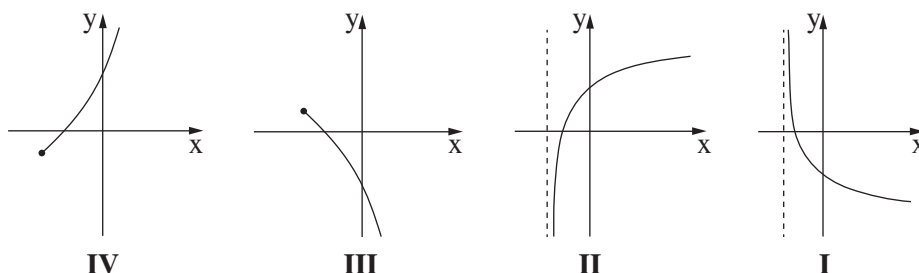
Given : The equation of the horizontal asymptote of function $g(x)$ is $y = 1$.

- ד. (1) Find k .
- (2) What are the coordinates of the extremum of function $g(x)$, and what type of extremum is it?

7. Consider the given function $f(x) = x - 2\sqrt{x+a}$, a is a parameter.

The graph of function $f(x)$ intersects the x -axis at the point $(6, 0)$.

- א. Show that $a = 3$.
- ב. Find the domain [תחום ההגדרה] of function $f(x)$.
- ג. Find the coordinates of **all** the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ד. Sketch a graph of the function $f(x)$.
- ה. One of the graphs I-IV below describes the graph of the derivative function $f'(x)$. Determine which one it is and explain your answer.
- ו. Calculate the area of the region bounded by the graph of the derivative function $f'(x)$, the line $x = 1$, and the x -axis.



8. Consider the given functions $f(x) = -x^2 + 9x$, $g(x) = x^2$.

Point A is located on the graph of function $f(x)$ in the first quadrant, above the graph of function $g(x)$.

Two lines are drawn from Point A :

one line which is perpendicular to the y-axis, intersecting it at Point C ,

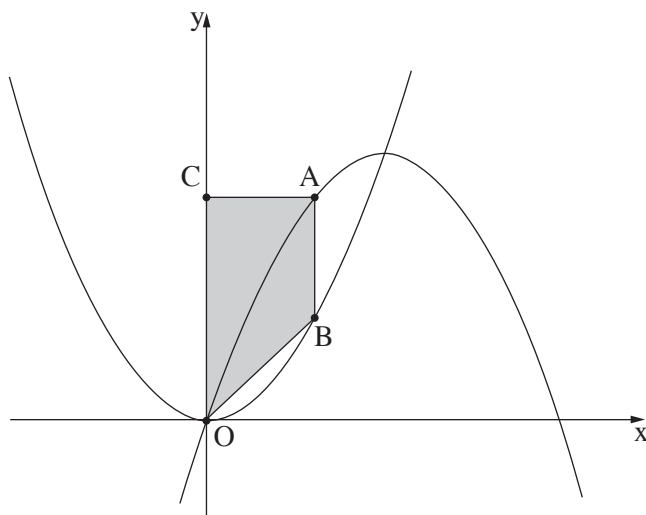
and one line which is parallel to the y-axis and intersects the graph of function $g(x)$ at Point B (see figure).

Point O is the origin.

Let t denote the x-coordinate of Point A .

א. Express the lengths of Segments AC , CO and AB , in terms of t .

ב. Find the value of t for which the area of Trapezoid ABOC is maximum (greatest).



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך