

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

3 Points — 3rd Exam

מתמטיקה

3 יחידות לימוד — שאלון שלישי

Instructions

א. Duration of the exam: Two hours

הוראות

א. משך הבחינה: שעותיים.

ב. Exam structure and breakdown of points:

This exam has six questions on the following topics:
Algebra, Differential and Integral Calculus.
Answer four questions.
Each question is worth 28 points.
The maximum number of points you can earn is 100.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שש שאלות בנושאים:
אלגברה, חשבון דיפרנציאלי ואינטגרלי.
יש לענות על ארבע שאלות – לכל שאלה 28 נקודות.
סך הנקודות לא יעלה על 100.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached)
- (3) A Hebrew-foreign language/ foreign language-Hebrew dictionary

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of your exam.

ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write in the answer booklet only. Write "טיוטה" at the top of every page you use for rough work.
If you use pages outside the answer booklet for rough work, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer four of the questions 1-6 (each question is worth 28 points).

If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Algebra

1. Yosef, owner of a media equipment store, bought loudspeakers and chargers from a factory.

He bought 23 fewer loudspeakers than chargers.

Yosef bought a total of 85 loudspeakers and chargers.

א. Find how many loudspeakers and how many chargers Yosef bought.

The original price of a loudspeaker is 2 times higher than the original price of a charger.

For each loudspeaker he bought, Yosef received a 20% discount off the original price, and for each charger he bought, he received a 15% discount off the original price.

Yosef paid a total of 6,589.5 shekels for the whole purchase.

Let x denote the price of a charger before the discount (the original price).

ב. Express in terms of x the price of a loudspeaker after the discount and the price of a charger after the discount.

ג. (1) Find the price of a loudspeaker and the price of a charger before the discount (the original price).

(2) Find the overall percentage discount that Yosef received on this purchase.

2. Given: Triangle ABC .

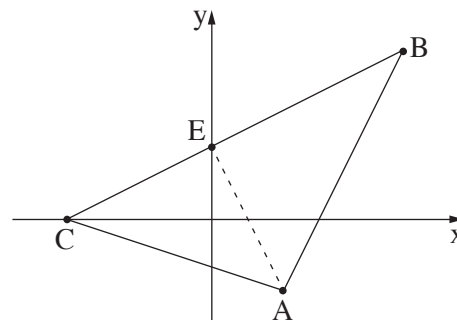
The equation of Side AC is $y = -\frac{1}{3}x - 2$.

The equation of Side AB is $y = 2x - 9$.

Vertex C is located on the x-axis (see figure).

א. Find the coordinates of Vertex C .

ב. Find the coordinates of Vertex A .



Given: The x-coordinate of Vertex B is 8 .

ג. Find the y-coordinate of Vertex B .

ד. Find the equation of Side BC .

Point E is the intersection point of Line BC and the y-axis.

ה. (1) Find the coordinates of Point E .

(2) Show that Line AE is perpendicular to Side BC .

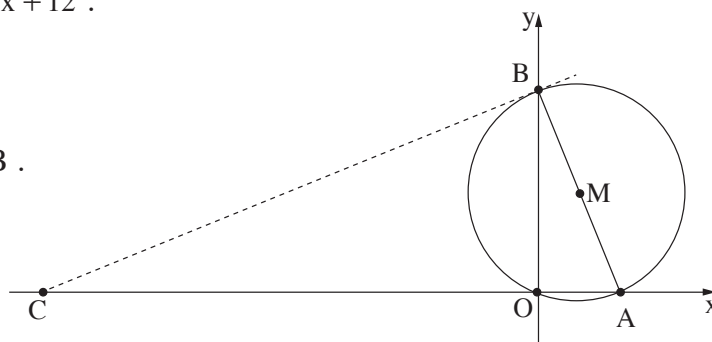
ו. Calculate the area of Triangle ACE .

3. Given: Line AB , whose equation is $y = -3x + 12$.

This line intersects the x -axis at Point A
 and the y -axis at Point B (see figure).

א. Find the coordinates of Points A and B .

Given: AB is a diameter of a circle whose
 center is M .



ב. Find the coordinates of Point M .

ג. (1) Find the length of the circle's radius.

(2) Write out the equation of the circle.

A tangent to the circle was drawn through Point B .

ד. (1) Find the slope of the tangent.

(2) Find the equation of the tangent.

The tangent intersects the x -axis at Point C .

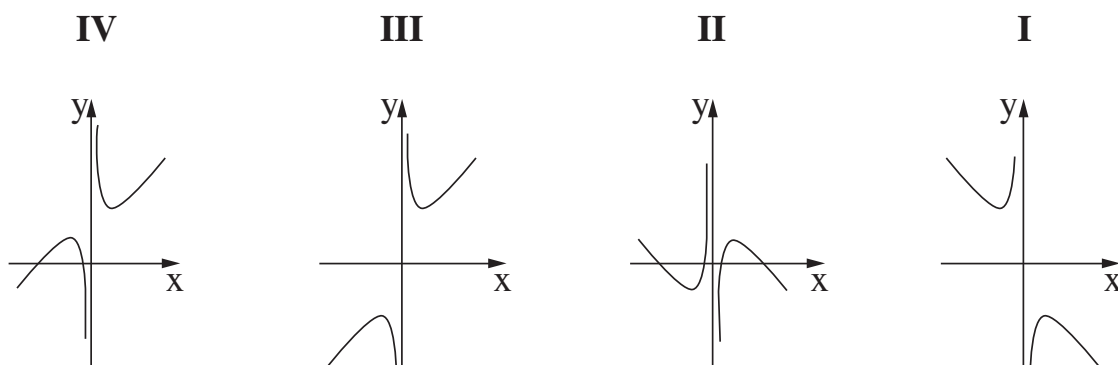
ה. (1) Find the coordinates of Point C .

(2) Find the area of Quadrilateral $BCOM$ (O is the origin).

Differential and Integral Calculus

4. Given: the function $f(x) = \frac{25}{x} + 4x$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x -axis.
- ב. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ג. One of the graphs I-IV at the end of the question describes the graph of function $f(x)$. Determine which one it is.
- ד. Give an example of the coordinates of a point on the graph of function $f(x)$ at which the slope of the tangent to the graph of the function is positive.



5. Given: the function $f(x) = x^3 - 12x + 2$.

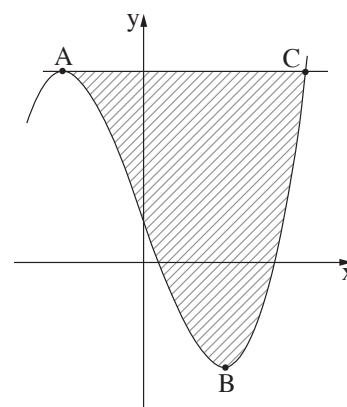
Function $f(x)$ has two extrema [נקודות קיצון]:

Point A is the maximum point and

Point B is the minimum point (see figure).

א. Find the coordinates of Points A and B.

ב. Find the equation of the tangent to the graph of function $f(x)$ at Point A.



The tangent whose equation you found for Item ב intersects the graph of function $f(x)$ at Point C.

ג. Show that the coordinates of Point C are (4, 18).

ד. Find the area of the shaded region in the figure:
the area bounded by the graph of function $f(x)$ and the tangent.

6. The figure to the right shows the graphs of two functions:

$$f(x) = x^2 - 2x + 6,$$

$$g(x) = -x^2 + 3x - 1.$$

Point A is located on the graph of function $f(x)$ in the first quadrant,

and Point B is located on the graph of function $g(x)$, such that

Segment AB is parallel to the y-axis.

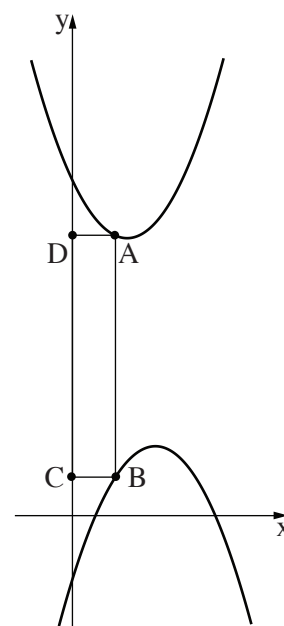
Points C and D are located on the y-axis, such that Quadrilateral ABCD is a rectangle, as shown in the figure.

Let x be the x-coordinate of Point A.

א. (1) Express the coordinates of Points A and B in terms of x .

(2) Express the length of Segment AB in terms of x .

ב. Find the x-coordinate of Point A for which the perimeter of Rectangle ABCD is smallest (minimal).



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך