

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

$$2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ points}$$

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions

$$1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.

Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

$$2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ נקודות}$$

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

$$1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ נקודות}$$

$$\text{סך הכול} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה מסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיטה" בראש כל עמוד המשמש טיטה.

כתיבת טיטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The point $(t, 0)$ is the focus of a parabola in canonical form [פרבולה קנונית] and the focus of an ellipse in canonical form [אליפסה קנונית]. t is a positive parameter.

The length of the major axis of the ellipse is $4t$.

In the items below, express your answers in terms of t , if necessary.

- א. Find the equation of the parabola and the equation of the ellipse.

The directrix of the parabola intersects the ellipse at two points, A and B . Point A is located above Point B .

- ב. Find the coordinates of Points A and B .

A line perpendicular to the x -axis passes through the focus of the parabola and intersects the parabola at two points, C and D .

Point C is located above Point D .

- ג. Find the coordinates of Points C and D .

- ד. (1) Explain why the four points A , B , C , and D are located on one circle.

- (2) Find the coordinates of the center of this circle.

2. Two planes are given: $\pi_1: z - 3 = 0$, $\pi_2: ay + z - 8 = 0$. a is a parameter that is not 0 .

The angle between the two planes is 45° .

- א. Find the possible values for the parameter a .

The point $A(2, -2, 6)$ is located on one of the given planes. A perpendicular was dropped from Point A to the other plane. This perpendicular intersects the other plane at Point B .

- ב. Find the length of Segment AB .

ℓ is the line of intersection between the two planes π_1 and π_2 .

- ג. Find the parametric representation of ℓ .

A perpendicular was dropped from Point B to the line ℓ . The perpendicular intersects the line ℓ at Point C .

- ד. Find the area of Triangle ABC (you may sketch the triangle on a set of axes to find the area).

3. Given: two complex numbers:

$$z_1 = (2a^2 + 5a + 4) + (2a^2 + 3a + 2)i$$

$$z_2 = (a^2 + 8a + 8) + (2 - a^2 + 2a)i$$

a is a real parameter.

א. Find the value of a for which the numbers z_1 and z_2 are conjugates.

Substitute the value of a that you found, and answer the items ב-ג.

The following numbers are given: $w_1 = \left(\frac{z_1}{\sqrt{2}}\right)^{4n}$, $w_2 = \left(\frac{z_2}{\sqrt{2}}\right)^{4n+2}$. n is a natural number.

ב. Prove that for any natural n :

(1) The number w_1 is a real number.

(2) The number w_2 is a pure imaginary number.

ג. Given: the equation $|z - p| = m$. p and m are real parameters; z is a complex number.

For what values of p and m does the given equation describe a circle on the Gauss plane on which the numbers w_1 and w_2 are located for any natural n ? Explain.

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: the function $f(x) = \frac{e^{2x} - 3e^x + m}{4}$. m is a parameter.

We know that the line $y = -1$ is an asymptote of the function $f(x)$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
(2) Find m .
(3) Find the coordinates of the points of intersection of the graph of the function $f(x)$ with the axes.
(4) Find the coordinates of the extrema [נקודות קיצון] of the function $f(x)$, and determine what type of extrema they are (if there are any).

- ב. Sketch a graph of the function $f(x)$.

Given: function $g(x) = \frac{1}{f(x)} + 1$.

- ג. (1) Find the domain of the function $g(x)$.
(2) Find the equations of the asymptotes of the function $g(x)$ that are perpendicular to the axes.

- ד. Sketch a graph of the function $g(x)$.

Given: a parameter t on the interval $0 < t < \ln 4$.

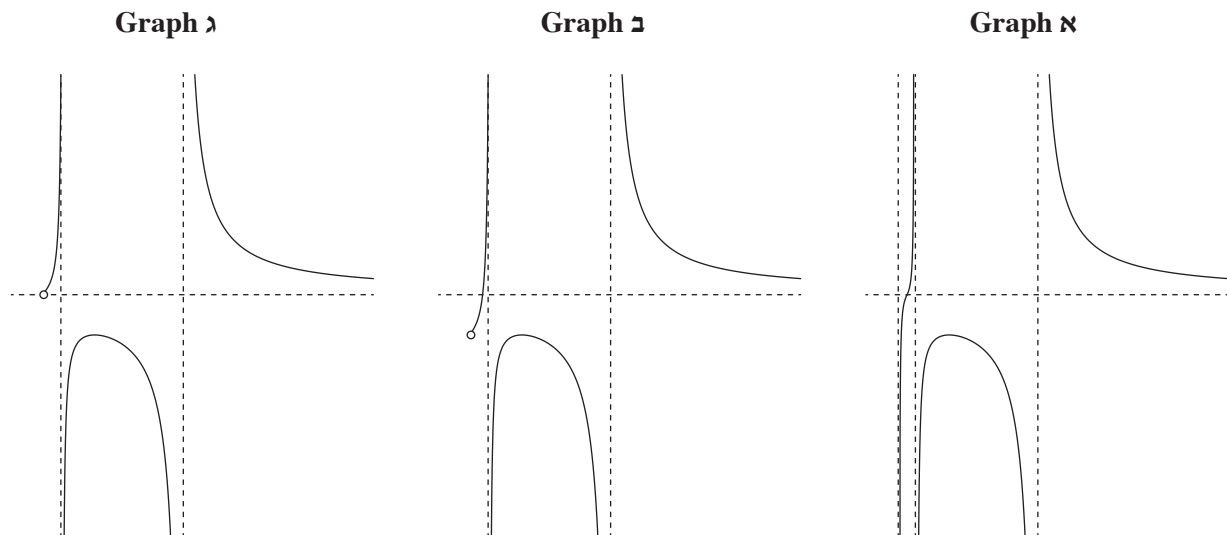
- ה. Find the value of t for which the value of the expression $\int_0^t g(x) dx$ is greatest [מקסימלי].
Explain your answer.

5. Given: the function $f(x) = \frac{(\ln(x))^2}{(\ln(x))^2 - 1}$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes.
- (3) Find the intervals on which the function $f(x)$ is increasing and on which it is decreasing.

- ב. Below are three graphs on which the x-axis and y-axis are not marked. The dotted lines represent the asymptotes of the function $f(x)$ that are perpendicular to the axes, and the empty circle represents a point at which the function is not defined.

Determine which of the graphs represents the graph of the function $f(x)$, copy this graph into your answer booklet, and add the axes. Explain your reasoning.



- ג. (1) Is there a solution to the equation $f(x) = 1$? Explain your answer.
- (2) What is the value of k for which the equation $f(x) = k$ has only one solution? Explain your answer.

Given: the functions $h(x) = (\ln(x))^2 + 1$, $g(x) = \frac{1}{f(x) - 1}$.

Let ABCD be a rectangle. Points A and B are two points on the x-axis at which the function $g(x)$ is not defined. Points C and D are located on the graph of the function $h(x)$.

- ד. What is the area of Rectangle ABCD? Explain your answer.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך