The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2022
Exam Number: 035581
Appendix: Formula sheets for 5 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשפ"ב, 2022
מספר השאלון: 035581
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

Note: This exam has special instructions. Answer the questions according to these instructions.

שים לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics
5 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts and a total of eight questions.
Part One: Algebra and Probability
Part Two: Planar Geometry and Planar Trigonometry
Part Three: Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions
Answer five questions of your choice —
5×20 = 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator.
Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or may lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה
5 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון: אלגברה והסתברות
פרק שני: גאומטריה וטריגונומטריה במישור
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות

עליך לענות על חמש שאלות לבחירתך — 5×20 = 100 נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Answer five of the questions 1-8 (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra and Probability

1. Three swimmers – Eitan, Gal and Yaakov – train in a 50-meter-long swimming pool.
Each swimmer begins swimming from the starting point of the pool [תחילת הבריכה], swims to the end of the pool [סוף הבריכה], and immediately turns round and swims back to the starting point. Each of the swimmers swims at a constant speed.
On Sunday, each of the three swimmers began his swim at a different time.
Gal began swimming 10 seconds after Eitan.
Yaakov began swimming 15 seconds after Eitan.
15 seconds after Yaakov began swimming, all the swimmers had covered the same distance from the starting point, but had not yet reached the end of the pool.
As soon as Gal reached the end of the pool, he turned round and began swimming back toward the starting point. On his way back, 4 meters away from the end of the pool, he met Eitan.

א. Calculate the speed of each of the three swimmers.

ב. When Eitan and Yaakov met up for the second time, how many meters were they from the end of the pool?

On Monday, Gal and Yaakov began swimming from the starting point at the same time, and each of them swam at the same speed at which he had swum on Sunday. When each of them reached the end of the pool, he immediately turned round and swam back to the starting point. When he reached the starting point, he turned round again and swam toward the end of the pool, and so on. Both swimmers stopped swimming when they met up at the starting point.

ג. How many meters did Yaakov swim that day?

/Continued on page 3/

2. Given: an increasing arithmetic sequence A, whose terms are $a_1, a_2, a_3, \ldots$; the common difference of this sequence is d.

S_n denotes the sum of the first n terms of the sequence A, for any natural n.

An additional sequence, B, whose terms are $b_1, b_2, b_3, \ldots$, is defined.

The terms of the sequence B satisfy $b_n = S_{n+1} - S_n$ for any natural n.

א. **(1)** Is the sequence B an arithmetic sequence? Explain.

(2) Is the sequence B identical to the sequence A? Explain.

T_n denotes the sum of the first n terms of the sequence B, for any natural n.

ב. Prove that any natural even n satisfies the following:

$$T_n = \frac{(b_1 + b_2)(b_1 - b_2) + (b_3 + b_4)(b_3 - b_4) + \ldots + (b_{n-1} + b_n)(b_{n-1} - b_n)}{-d}$$

Given: $b_1^2 - b_2^2 + b_3^2 - b_4^2 + \ldots + b_{39}^2 - b_{40}^2 = -95$

$T_5 = -20$

ג. Calculate b_1 and d (you may use Item ב to help you).

The terms in the odd positions in the sequence A are added up one after the other, starting from the first term.

ד. What is the minimum number of terms that need to be added up in this way for the sum to be a positive integer [חיובי שלם]? Explain.

3. A box contains three strawberry-flavored candies and two mint-flavored candies. Lior takes a candy out of the box at random. If the candy is mint-flavored, he returns it to the box, and if it is strawberry-flavored, he eats it immediately.

א. Lior takes three candies out of the box one after the other in the manner described above.

(1) Calculate the probability that Lior will eat exactly one candy.

(2) Calculate the probability that Lior ate the second candy he took out, if we know that Lior ate exactly one candy.

ב. Lior takes n candies out of the box one after the other in the manner described above.

Express in terms of n the probability that Lior will eat at least one candy.

ג. Lior received two boxes of candies, each identical to the box described above.

Lior takes three candies out of each of the two boxes in the manner described above.

Calculate the probability that Lior will eat exactly three candies, all three from the same box.

/Continued on page 4/

Part Two — Planar Geometry and Planar Trigonometry

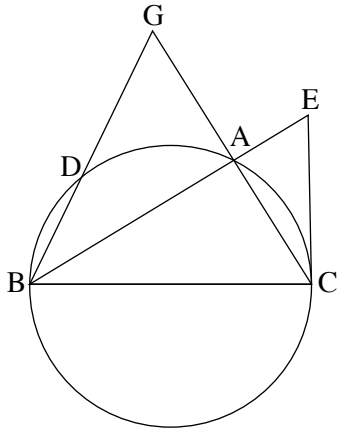
4. Triangle ABC is inscribed in a circle whose radius is R (see figure).

Side BC is a diameter of the circle.

AG is an extension of Side CA .

Segment GB intersects the circle at Point D .

Given: $GA = AC$.



א. Prove that the line AB bisects $\sphericalangle GBC$.

ב. Prove that $\triangle GBC \sim \triangle GAD$.

Given: $\frac{S_{DBCA}}{S_{GAD}} = 15$.

ג. Express the length of Side AC in terms of R .

A tangent to the circle was drawn through Point C , intersecting the extension of Segment BA at Point E .

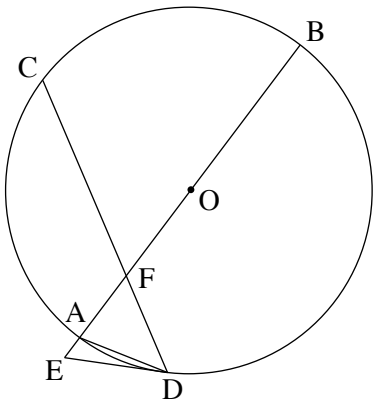
ד. Calculate by what factor [פי כמה] the area of Triangle CBE is greater than the area of Triangle ABC.

5. AB is a diameter in a circle whose radius is R and center is O .

Chord CD intersects the diameter AB at Point F .

The tangent to the circle at Point D intersects the extension of the diameter AB at Point E (see figure).

Let $\sphericalangle ADE = \alpha$.



א. Show that $\sphericalangle BAD = 90^o - \alpha$.

Given: $ED = FD$.

ב. Express the size of $\sphericalangle CDA$ in terms of α .

ג. Express the area of Triangle AFD in terms of R and α .

ד. **(1)** Express the ratio of the areas $\frac{S_{AFD}}{S_{AED}}$ in terms of α .

(2) Given: $\frac{S_{AFD}}{S_{AED}} = 1 + \sqrt{3}$.

Find α .

/Continued on page 5/

Part Three—Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

6. Given: the function $f(x)=\frac{x^2}{(x^3-m)^2}$, m is a positive parameter.

א. Express your answers in terms of m , if necessary.

(1) Find the domain [תחום ההגדרה] of function f(x) .

(2) Find the equations of the asymptotes of function f(x) that are perpendicular to the axes.

We know that function f(x) has an extremum [נקודת קיצון] at the point at which $x=(-1)$.

ב. Find the value of m .

Substitute this value for m in function f(x) , and answer Items ג–ה.

ג. Find the coordinates of the extrema [נקודות קיצון] of function f(x) , and determine what type of extrema they are.

ד. Sketch a graph of the function f(x) .

ה. Given: the function $g(x)=k\cdot f(x)$, k is a negative parameter.

(1) Sketch a possible graph of the function g(x) .

(2) A perpendicular to the x-axis is drawn through the left-hand extremum of g(x) .
Given: The area of the region bounded by the perpendicular, the graph of the function g(x) , and the x-axis is 1 (the region to the right of the perpendicular).
Find the value of k .

/Continued on page 6/

7. Given: The function $f(x) = 3x + 2 \cdot \sqrt{x^2 - 2x}$.

א. **(1)** Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the domain of the derivative function $f'(x)$.

(3) Find the equations of the asymptotes of the derivative function $f'(x)$ that are perpendicular to the axes.

(4) Find the coordinates of the intersection point of the graph of the derivative function $f'(x)$ with the x-axis.

Your answer should be correct to two decimal places.

(5) Sketch a graph of the derivative function $f'(x)$, if we know that the derivative function $f'(x)$ has no extrema [נקודות קיצון].

ב. **(1)** Find the coordinates of the extrema of function $f(x)$, and determine what type of extrema they are.

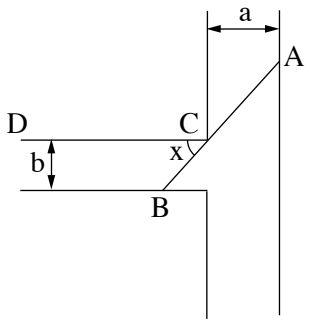
(2) Sketch a graph of function $f(x)$.

ג. Could a line whose equation is $y = 4x + c$ (c is a parameter) be tangent to the graph of function $f(x)$? Explain.

8. A main canal, whose width, a , is constant, is joined perpendicularly to a side canal whose width, b , is constant.

Point C is the point at which a wall of the main canal meets a wall of the side canal (see figure).

An engineer is planning a straight dam that starts at Point A on a wall of the main canal, passes through Point C , and reaches Point B on a wall of the side canal.



The dam will form an angle of size x with the wall CD of the side canal, as shown in the figure.

א. Express the length of the dam AB in terms of a , b , and x .

Given: $a = 2b$.

ב. Find the value of x for which the length of the dam AB is smallest (minimum).

ג. We know that the minimum length of the dam is 8 . Find b .

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך