

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes.

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. Exam structure and breakdown of points:

This exam has two parts.

ב. מבנה השאלון ומפתח ההערכה:

Part One – Sequences, Spatial Trigonometry

בשאלון זה שני פרקים.

$$- 1 \times 33 \frac{1}{3} - 33 \text{ points}$$

פרק ראשון – סדרות, טריגונומטריה במרחב

$$- 33 \frac{1}{3} - 33 \frac{1}{3} \times 1 - \text{נקודות}$$

Part Two – Growth and Decay, Differential and Integral

פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי

Calculus of Trigonometric Functions,

של פונקציות טריגונומטריות,

Exponential and Logarithmic Functions,

פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה

and Power Functions

$$- 2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ points}$$

$$- 66 \frac{2}{3} - 33 \frac{1}{3} \times 2 - \text{נקודות}$$

Total – 100 points

סך הכול – 100 נקודות

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator.

(1) מחשבון לא גרפי.

Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) Formula sheets (attached).

(2) דפי נוסחאות (מצורפים).

(3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

(3) מילון עברי-לועזי / לועזי-עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) Start each question on a new page.

(2) התחל כל שאלה בעמוד חדש.

Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Lack of detail may lower your score or lead to the disqualification of your exam.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you use any draft paper outside the answer booklet your exam may be disqualified.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Sequences, Spatial Trigonometry ($33\frac{1}{3}$ points)

Answer one of the questions 1-2.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

Sequences

1. a_n is an arithmetic sequence.

Given: $a_2 + a_4 = 124$.

- א. Find a_3 .

Given: $a_4 = 76$.

- ב. Find a_1 and the common difference of this sequence.
ג. (1) Express the term a_n in terms of n .
(2) Explain why all the terms in the sequence a_n are even.

The given sequence a_n contains 64 terms.

- ד. Every third term ($a_3, a_6, \dots$) in the given sequence was deleted.
What is the sum of the remaining terms?

Spatial Trigonometry

2. The figure on the right describes a cuboid $ABCD A' B' C' D'$ with a base $ABCD$ that is a rectangle.

Given: $AD' = 12$;

the size of the angle between AD' and the base $ABCD$ is 60° .

- א. Find the length of the cuboid height.

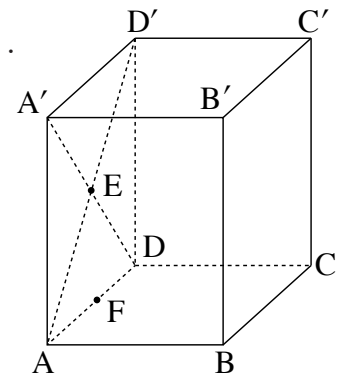
Given: The volume of the cuboid is 432.

- ב. Find the lengths of the sides of the base $ABCD$.

Point E is the intersection point of the diagonals AD' and $A'D$.

Point F is the midpoint of the side AD (see figure).

- ג. (1) Find the length of the segment BF .
(2) Find the size of the angle between EB and the cuboid's base, $ABCD$.



**Part Two — Growth and Decay, Differential and Integral Calculus of
Trigonometric Functions, Exponential and Logarithmic Functions,
and Power Functions ($66\frac{2}{3}$ points)**

Answer two of the questions 3-5 (each question — $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

3. Consider the given functions: $g(x) = \cos(2x)$, $f(x) = \cos(x)$, which are defined on the domain $0 \leq x \leq \pi$.

- א. Find the coordinates of the intersection points of the functions $f(x)$ and $g(x)$.
- ב. (1) Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$ and determine what type of extrema they are.
(2) Find the coordinates of all the extrema of function $g(x)$ and determine what type of extrema they are.
- ג. Draw graphs of the functions $f(x)$ and $g(x)$ on the same set of axes.

$a > 0$ is a parameter.

Given: The area of the region bounded by the graph of the function $a \cdot f(x)$ and the graph of the function $a \cdot g(x)$ between their two points of intersection is $3\sqrt{3}$.

ד. Find a .

4. Consider the given function $f(x) = \frac{8}{e^x} + \frac{e^x}{2} + c$, c is a parameter.

א. Find the domain [תחום ההגדרה] of function $f(x)$.

Given: The graph of function $f(x)$ passes through the origin.

ב. Find c .

Substitute the value of c which you found in Item ב in function $f(x)$, and answer Items ג-ה.

- ג. Find the coordinates of the other point of intersection of the graph of function $f(x)$ with the x -axis.
- ד. Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$ and determine what type of extremum it is.
- ה. Sketch a graph of function $f(x)$.

Let S denote the area of the region bounded by the graph of function $f(x)$ and the x -axis.

Let S_1 denote the area of the region bounded by the graph of function $-f(x)$ and the x -axis.

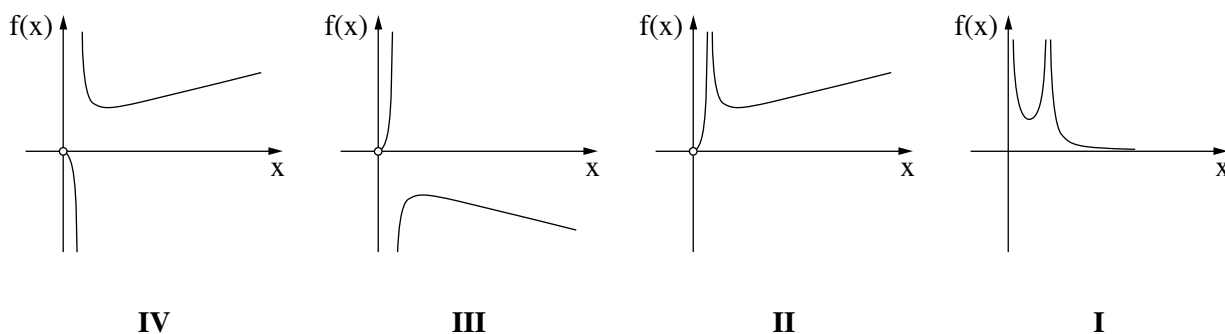
Let S_2 denote the area of the region bounded by the graph of function $2 \cdot f(x)$ and the x -axis.

- ו. For each of the areas S_1 and S_2 , determine whether it is larger than S , smaller than S , or equal to S .

Explain your answers.

5. Consider the given function $f(x) = \frac{4x}{1 + \ln(2x)}$.

- א. Find the domain [תחום ההגדרה] of function $f(x)$.
- ב. (1) Does the graph of function $f(x)$ intersect the x-axis? Explain.
(2) Find the vertical asymptote of function $f(x)$.
- ג. Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$ and determine what type of extremum it is.
- ד. Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
- ה. One of the graphs I-IV below matches the graph of function $f(x)$. Determine which one it is, and explain your answer.
- ו. Find the interval on which $f(x) \cdot f'(x) < 0$ is satisfied.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך