

Note: There are special instructions in this exam. Please answer the questions according to these instructions.

שים לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam consists of three parts, with a total of eight questions.

Part One – Algebra, Analytical Geometry, Probability

Part Two – Planar Geometry and Planar Trigonometry

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer **five** questions of your choice – $5 \times 20 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד – שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון – אלגברה, גאומטריה אנליטית, הסתברות
פרק שני – גאומטריה וטריגונומטריה במישור
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

עליך לענות על חמש שאלות לבחירתך – $5 \times 20 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיטה" בראש כל עמוד המשמש טיטה. כתיבת טיטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

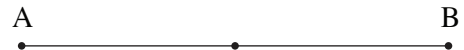
Answer five of the questions 1-8 (each question — 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry, Probability

1. Ori and David ran along a straight track, AB . Each ran at a fixed speed.

On **Sunday** they set off together from the middle of Track AB :



Ori ran from the middle of the track to Point B , and David ran from the middle of the track to Point A .

Ori arrived at Point B 40 minutes after setting off and David arrived at Point A one hour after setting off.

Let x denote David's speed (in km/h).

⌘. Express Ori's speed in terms of x .

On **Monday** they set off together from Point C on Track AB :



Ori ran from Point C to Point B , and David ran from Point C to Point A .

The length of AC is 4.5 km.

Ori reached Point B half an hour after David had reached Point A .

Each of them ran at the same speed as on Sunday.

Ⓐ. Find the length of Track AB .

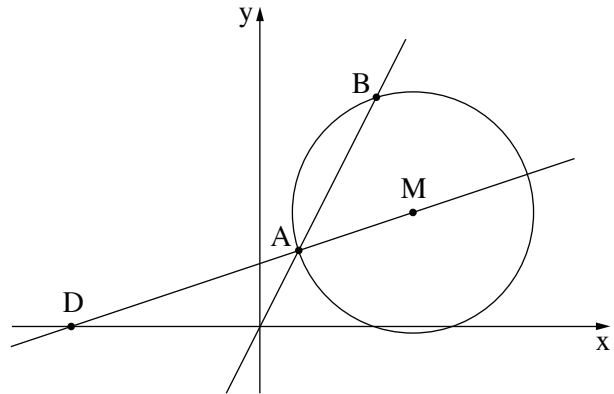
2. Point M is the center of a circle whose equation is $(x - 4)^2 + (y - 3)^2 = 10$.

The line $y = 2x$ intersects the circle at Points A and B, as shown in the figure below.

- א. Find the coordinates of Points A and B.

The line AM intersects the x-axis at Point D (see figure).

- ב. Find the coordinates of Point D.
ג. Prove that BM is perpendicular to DM.
ד. Is the line DB tangent to the circle? Explain.
ה. Calculate the area of Triangle ABD.



3. A community center offers only two classes: a soccer class and a tennis class. Participants may **only** join **one** of these two classes.

The total number of boys participating in the two classes is the same as the total number of girls participating in them.

80% of the boys are in the soccer class.

The number of girls in the tennis class is 3 times greater than the number of girls in the soccer class.

A random participant is selected.

- א. What is the probability that the participant selected is a boy from the soccer class?
ב. If we know that the participant selected is in the tennis class, what is the probability that this participant is a boy?

We know that the total number of participants in the two classes together is 200.

- ג. (1) How many participants are there in total in the tennis class?
(2) Of all the participants in the classes, two are selected at random, one after the other (without replacement).

What is the probability that they are both in the tennis class?

Your answer must be correct to 3 decimal places.

Part Two — Planar Geometry and Planar Trigonometry

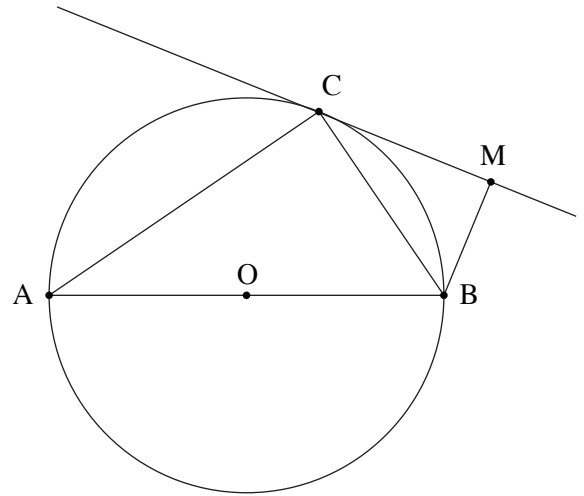
4. The figure on the right describes a circle whose center is Point O .

AB is a diameter of the circle.

A tangent to the circle is drawn through Point C ;

Point C is on the circle.

A perpendicular is dropped from Point B to the tangent. This perpendicular intersects the tangent at Point M , as shown in the figure.



א. Prove that $\angle MBC = \angle CBA$.

ב. Prove that $BC^2 = AB \cdot BM$.

A line, parallel to BC , is drawn through the center of the circle, O .

This line intersects AC at Point E .

Given: The area of Triangle ABC is 3.24 times greater than the area of Triangle CBM ;

$$OE = 5.$$

ג. Find AB .

5. The figure below describes a triangle BDM and a triangle ABC ;

Point C is the midpoint of Side BD .

Given: The area of Triangle ABC is 27.8;

$$AB = 10;$$

$$\angle BAC = 44^\circ.$$

א. Find the length of Side AC .

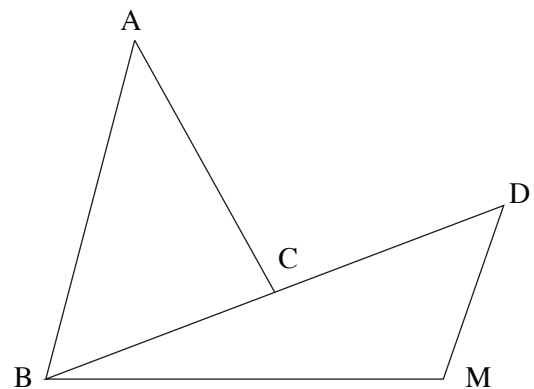
ב. Find the length of Side BC .

Given: $BM = 11.4$;

$$\angle BDM = 50^\circ;$$

$$\angle BMD > 90^\circ.$$

ג. Find the size of Angle BMD .



The bisector of Angle BAC is drawn from Point A , intersecting Side BM at Point F .

ד. Find the angles of Triangle AFB .

Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the given function: $f(x) = \frac{4}{4x^2 - 1} + b$. b is a parameter.

א. Find the domain [תחום ההגדרה] of function $f(x)$.

ב. (1) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$ and determine what type of extremum it is (express in terms of b , if necessary).

(2) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.

Given: The line $y = -2$ is tangent to function $f(x)$ at its extremum.

ג. Find b .

Substitute b , which you found in Item ג, in function $f(x)$ and answer Items ד-ו.

ד. (1) Find the asymptotes of function $f(x)$ that are perpendicular to the axes.

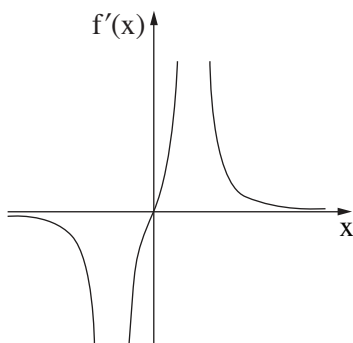
(2) Sketch a graph of function $f(x)$.

$f'(x)$ is the derivative function of function $f(x)$.

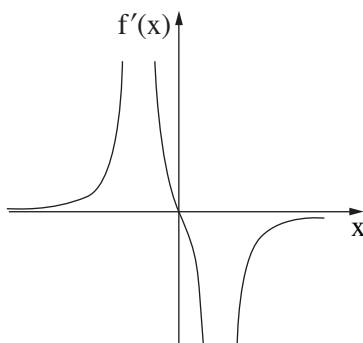
ה. One of the graphs I-III below describes the derivative function $f'(x)$.

Determine which one it is, and explain your answer.

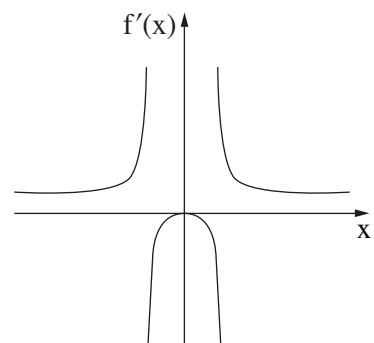
ו. Find the area of the region bounded by the graph of the derivative function $f'(x)$, the line $x = \frac{1}{3}$, and the x -axis.



III



II



I

7. Consider the given function $f(x) = x \cdot \sqrt{x + 18}$.

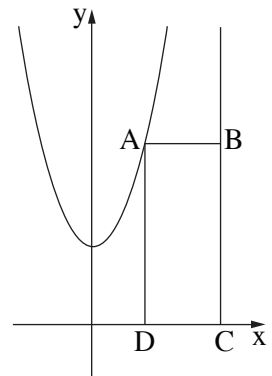
- א. Find the domain [תחום ההגדרה] of function $f(x)$.
- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
- ג. Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
- ד. Sketch a graph of function $f(x)$.
- ה. Consider the given function $g(x) = -2 \cdot f(x)$.
 - (1) Find the coordinates of all the extrema of function $g(x)$, and determine what type of extrema they are.
 - (2) Let A and B denote the internal extrema of the functions $f(x)$ and $g(x)$, respectively. O is the origin. Calculate the area of Triangle ABO.

8. Consider the given function $f(x) = x^2 + 3$ and the given line $x = 5$.

Point A is located on the graph of function $f(x)$ in the first quadrant to the left of the line. A line is drawn from Point A parallel to the x-axis and intersecting the given line at Point B. Points C and D are located on the x-axis such that the quadrilateral ABCD is a rectangle (see figure).

Let t denote the x-coordinate of Point A.

- א. Find the value of t for which the area of Rectangle ABCD is greatest (maximum).
- ב. Could there be a Rectangle ABCD constructed as described above whose area is 30? Explain.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך