

Mathematics

3 Points – 3rd Exam

Instructions for examinees

א. Duration of the exam: Two hours

ב. Exam structure and breakdown of points:

This exam has six questions on the topics:
algebra, differential and integral calculus.

Answer four questions — $25 \times 4 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to disqualification of your exam.

Write in your answer booklet only. Write "טייטה" at the top of each draft page. If you write draft work outside your answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שתיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות — $25 \times 4 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות

במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון

עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי

הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה

ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טייטה" בראש כל עמוד המשמש טייטה.

כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer four of the questions 1-6 (each question — 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to disqualification of your exam.

Algebra

1. A family of two adults and three children stayed at a hotel for one night.

The price of one night at this hotel for an adult is 85 shekels more than for a child.

This family paid the full price for the adults, and received a 32% discount for the children.

The family paid 1,361.8 shekels in total for the night at the hotel.

- א. What is the price of one night at this hotel for an adult and what is the price for a child (without the discount)?
- ב. (1) What is the full price that this family would have paid without the discount?
(2) What discount did the family receive in total out of the full price, in percent?

2. Consider the given quadrilateral ABCD, in which the diagonal AC is perpendicular to the diagonal BD (see figure).

The diagonals of the quadrilateral meet at Point E.

The diagonal AC lies on a line whose equation is $y = -3x + 42$.

The x-coordinate of Vertex A is 7.

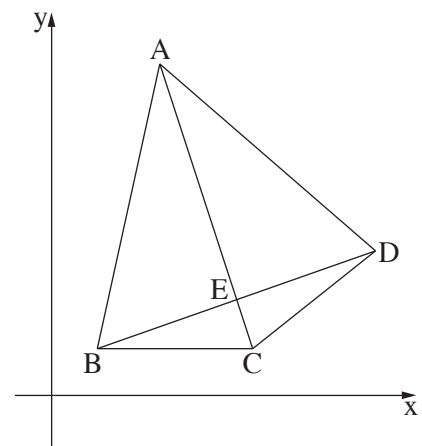
א. Find the y-coordinate of Vertex A.

Point E is the midpoint of Diagonal BD and its coordinates are (12, 6).

ב. Find the equation of Diagonal BD.

Given: The equation of the line on which Side AB lies is $y = 4.5x - 10.5$.

- א. (1) Find the coordinates of Vertex B.
(2) Find the coordinates of Vertex D.
- ב. Side BC is parallel to the x-axis.
(1) Find the coordinates of Vertex C.
(2) Is Triangle DBC an isosceles triangle? Explain.



3. The figure on the right describes a circle whose center, M , is in the first quadrant.

AC is a diameter of the circle.

Point C is located on the x -axis.

The chord BC lies on a line whose equation is

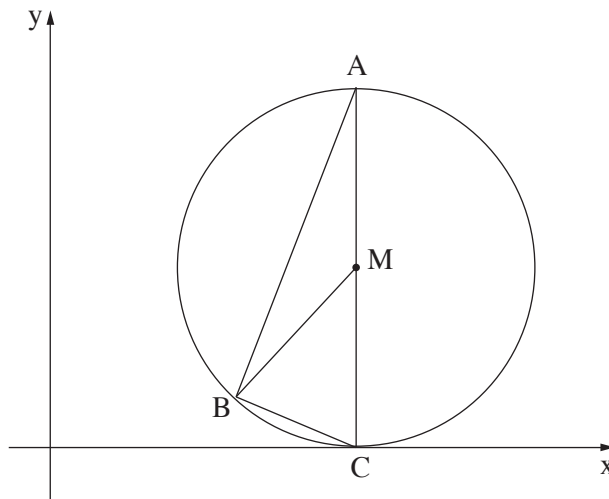
$$y = -\frac{2}{5}x + 10.$$

- א. Find the coordinates of Point C .

AC is perpendicular to the x -axis.

Given: $AC = 29$.

- ב. (1) Find the coordinates of Point A .
(2) Find the equation of the circle.



The chord AB is perpendicular to the chord BC .

- ג. (1) Find the equation of the line on which the chord AB lies.
(2) Find the coordinates of Point B .
(3) Find the area of Triangle BMC .

Differential and Integral Calculus

4. Consider the given function $f(x) = 4x + \frac{9}{x} - 20$.

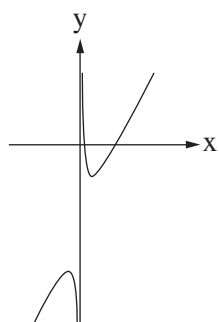
- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
- ב. Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and determine what type of extrema they are.
- ג. One of the graphs I-IV below describes the graph of the function $f(x)$. Determine which one it is, and explain your answer.
- ד. Consider the following four equations of lines which are parallel to the x-axis.
Which of the lines (1)-(4) intersects the graph of the function $f(x)$ at only one point?
Explain your answer.

1) $y = 0$

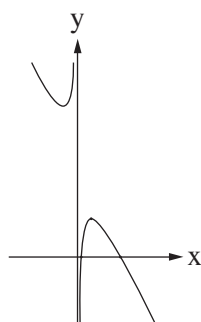
2) $y = -32$

3) $y = -40$

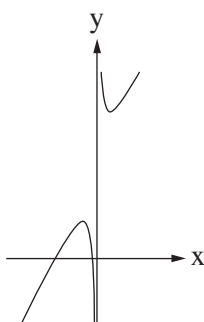
4) $y = 2$



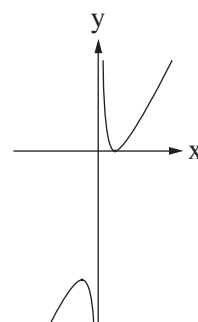
IV



III



II



I

5. Consider the given function $y = -x^2 + 6x - 5$.

The graph of the function intersects the x-axis at Points A and B, as shown in the figure on the right.

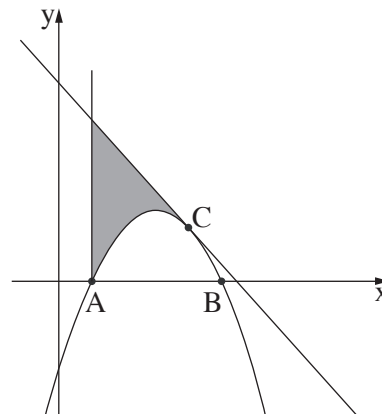
- א. Find the coordinates of Points A and B.

A tangent to the graph of the function is drawn through Point C, at which $x = 4$.

- ב. Show that the equation of the tangent is $y = -2x + 11$.

A perpendicular to the x-axis is drawn from Point A upward (see figure).

- ג. Find the area of the region bounded by the graph of the function, the tangent, and the perpendicular (the shaded region in the figure).



6. Consider a given rectangular plot whose dimensions are 60 meters by 80 meters, as shown in the figure below.

The plot is divided by two straight lines into four rectangular sections:

two sections for gardens (the shaded rectangles in the figure)

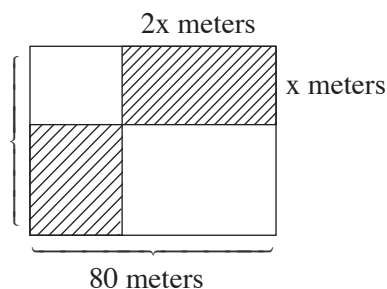
and two sections for building (the white rectangles in the figure).

The dimensions of the upper shaded rectangle are 60 meters by x meters by $2x$ meters, as shown in the figure.

- א. Express the dimensions of the lower shaded rectangle in terms of x .

- ב. Find the value of x for which the sum of the areas for gardens (the shaded sections) is the smallest [מינימלי].

- ג. For the value of x you found in Item ב, find the size of the area for building.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך