The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2021, ***Moed Bet***
Exam Number: 035582
Appendix: Formula sheets for 5 points
English Translation (3)

מועד ב

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"א 2021, **מועד ב**
מספר השאלון: 035582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:
This exam consists of two parts with a total of five questions.
Part One: Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers
Part Two: Growth and Decay, Power Functions, Exponential and Logarithmic Functions

Answer three questions of your choice
$3 \times 33\frac{1}{3} = 100$ points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים ובהם חמש שאלות.
פרק ראשון: גאומטריה אנליטית, וקטורים, טריגונומטריה במרחב, מספרים מרוכבים.
פרק שני: גדילה ודעיכה, פונקציות חזקה, פונקציות מעריכיות ולוגריתמיות
עליך לענות על שלוש שאלות לבחירתך
$3 \times 33\frac{1}{3} = 100$ נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה מסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Answer three of the questions 1-5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers

1. Consider the parabola equation $y^2 = 2ax$ and the circle equation $x^2 + y^2 - 2ax - 2x = 0$.
a is a parameter greater than 0.

א. Find the coordinates of the intersection points of the parabola and the circle. Express in terms of a, if necessary.

A line with a positive slope passes through two of the intersection points of the parabola and the circle.

ב. Find the equation of the line. Express in terms of a, if necessary.

A perpendicular to the line is dropped from the center of the circle. The length of the perpendicular is $2\sqrt{5}$.

ג. **(1)** Express the center and the radius of the circle in terms of a.

(2) Find a.

A new circle is defined, whose center is identical to the center of the given circle and whose radius is 2 less than the radius of the given circle.

ד. Find the equation of the locus [מקום גאומטרי] of all the points for which the length of the tangent from these points to the new circle is equal to their distance from the line $x = -4$.

Continued on page 3 ◀

2. Consider the triangle ABC (see figure).

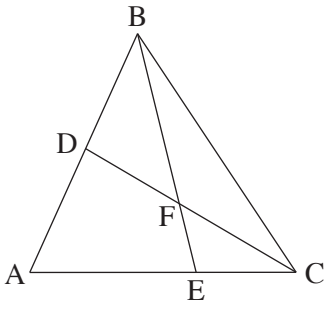
Point D is the midpoint of Side AB.

Point E divides Side AC according to the ratio $AE:EC=2:1$.

Point F is the intersection point of the segments BE and CD.

Let: $\overrightarrow{CB}=\underline{v}$, $\overrightarrow{CA}=\underline{u}$

k and t are numbers such that: $\overrightarrow{BF}=t\cdot\overrightarrow{BE}$, $\overrightarrow{CF}=k\cdot\overrightarrow{CD}$.



א. Find t and k.

The triangle ABC is on the plane $4x+2y+z-12=0$.

This plane intersects the x-axis at Point A, the y-axis at Point C, and the z-axis at Point B.

O is the origin.

ב. Find the coordinates of Points E and F.

ג. Find the equation of the plane AOE.

ד. Find the volume of the pyramid FAOE.

3. Consider Equation I: $z^4-2z^2+4=0$. z is a complex number.

א. Solve Equation I.

The solutions to the equation are represented by all the vertices of a polygon on the Gauss plane.

ב. Find the area of the polygon.

Consider Equation II: $(a\cdot z^2+b)(z+1)=0$. z is a complex number, a and b are real numbers that are not zero.

We know that two of the solutions to this equation are imaginary numbers [מספרים מדומים].

ג. Prove that $a\cdot b>0$.

ד. Find the solutions to Equation II. Express in terms of a and b, if necessary.

We know that the imaginary solutions to Equation II are represented by points on a circle whose center is at the origin and whose radius is larger by a factor of two than the absolute value of the solutions to Equation I.

ה. Find the ratio $\frac{b}{a}$.

Continued on page 4 ◀

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the function $f(x) = e^{(bx^2 - 2bx)} - 1$ which is defined for any x. $b < 0$ is a parameter.
Express your answers in terms of b, if necessary.

א. **(1)** Find the coordinates of the points of intersection of the graph of function $f(x)$ with the axes.

(2) Find the asymptotes of the function $f(x)$ that are parallel to the x-axis (if there are any).

(3) Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are (if there are any).

(4) Sketch a graph of function $f(x)$.

We define the function $g(x) = f(x + a)$, a is a parameter. We know that function $g(x)$ has an extremum [נקודת קיצון] on the y-axis.

ב. **(1)** Find a and express function $g(x)$ in terms of x and b.

(2) Is function $g(x)$ even, odd, or neither even nor odd? Explain.

(3) Sketch a graph of function $g(x)$.

ג. Find the x-coordinate of each of the extrema of the derivative function $g'(x)$ and determine what type of extrema they are.

ד. Insert $b = -0.5$ and calculate the area of the region bounded by the graph of the derivative function $g'(x)$, the x-axis, and the lines that pass through the extrema of the derivative function $g'(x)$ and are perpendicular to the x-axis.

Continued on page 5 ▶

5. Consider the given function $f(x)=a\cdot x^2-x^3$ which is defined for any x, a is a parameter.

Answer Items א-ג for $0<a$. Express your answers in terms of a, if necessary.

א. **(1)** Find the positive and the negative intervals of function $f(x)$.

(2) Sketch a graph of function $f(x)$.

Consider the function $g(x)=\ell n(f(x))$.

ב. **(1)** Find the domain [תחום ההגדרה] of function $g(x)$.

(2) Find the asymptotes of function $g(x)$ that are perpendicular to the axes (if there are any).

(3) Find the coordinates of the extremum [נקודת קיצון] of function $g(x)$, and determine what type of extremum it is.

ג. We know that the graph of function $g(x)$ has only one point of intersection with the x-axis.

(1) Sketch a graph of function $g(x)$.

(2) Find the range of possible values of a for which the graph of function $g(x)$ intersects the x-axis at one point only.

Answer Item ד for $a=0$.

ד. Sketch a graph of function $g(x)$. Mark on the graph the numerical values of the coordinates of the point at which the graph of the function intersects the x-axis.

Good Luck!

בהצלחה!