The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2021, ***Moed Bet***
Exam Number: 035581
Appendix: Formula sheets for 5 points
English Translation (3)

מועד ב

Note: There are special instructions in this exam.
Answer the questions according to these instructions.

Mathematics

5 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts with a total of eight questions.
Part One: Algebra and Probability
Part Two: Planar Geometry and Planar Trigonometry
Part Three: Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions
You must answer four questions of your choice
– $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or may lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"א, 2021, **מועד ב**
מספר השאלון: 035581
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

שים לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

מתמטיקה

5 יחידות לימוד - שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים ובהם שמונה שאלות.
פרק ראשון: אלגברה והסתברות
פרק שני: גאומטריה וטריגונומטריה במישור
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות
עליך לענות על ארבע שאלות לבחירתך
— $25 \times 4 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: **Explain <u>all</u> of your work, including calculations, clearly and in detail.**
Lack of detail may lower your score or lead to the disqualification of your exam.

Answer <u>four</u> of the questions 1-8 (each question – 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Part One — Algebra and Probability

1. Netta, Daniella and Roni practice walking and running on a 40 km route, AB .

At 8:00 Netta set out from Point A , walking at a speed of 4 km/h in the direction of Point B .

At 9:36 Daniella set out from Point B , running in the direction of Point A .

Two hours after Netta set out, Roni set out from Point B , running at a speed of 12 km/h in the direction of Point A .

Netta and Roni met and then continued on their respective ways.

An hour and 36 minutes after Netta and Roni met, Daniella reached Point A .

Each of the women maintained a constant speed throughout the entire practice session.

א. At what time did Netta and Roni meet?

ב. What is Daniella's running speed? Explain your answer.

ג. Did the three women all meet at any one point along the route? Explain your answer.

When each woman reaches the end of the route, she immediately turns around and heads back to her starting point.

ד. At what distance from Point B did Netta and Roni meet the second time?
Explain your answer.

/Continued on page 3/

2. Consider the given infinite geometric sequence a_n whose terms are a_1 , a_2 , a_3 , ... , and whose common ratio is q .

א. Express the values of the following sums in terms of a_1 and q .

(1) $A = a_2 + a_4 + a_6 + ... + a_{40}$

(2) $B = a_4 + a_8 + a_{12} + ... + a_{40}$

We know that a_n is an increasing sequence and that $\frac{A}{B} = \frac{10}{9}$.

ב. Find the value of q .

The sequence a_n is used to construct an infinite geometric sequence b_n which satisfies $b_n = 3 \cdot a_{n+1}$ for any natural n .

ג. Find the common ratio of the sequence b_n .

A new infinite geometric sequence is constructed: $-\frac{1}{b_1}$, $\frac{1}{b_2}$, $-\frac{1}{b_3}$, $\frac{1}{b_4}$, ...

ד. Express the sum of all the terms of the new sequence in terms of a_1 .

Given: the sequence $\frac{1}{a_1}$, a_1 , b_1 .

ה. **(1)** Could this be an arithmetic sequence? Explain your answer.

(2) Could this be a geometric sequence? Explain your answer.

3. A sports competition is held at a school and many students participate. Each participant must pass 3 consecutive obstacles in order of sequence. Participants who fail to pass an obstacle are immediately disqualified from the competition. The probability of passing an obstacle is different for each obstacle but the same for each participant. Participants who successfully pass all three obstacles qualify for the semi-finals.

28% of the participants succeeded in passing the first two obstacles. The probability that a participant who passed the first two obstacles would be disqualified was 3 times greater than the probability of the participant qualifying for the semi-finals.

א. Calculate the probability of a participant in the competition qualifying for the semi-finals.

The probability of a participant passing the first obstacle and failing to pass the second obstacle is 0.42 .

ב. Calculate the probability of a participant <u>failing</u> to pass the first obstacle.

ג. Three participants are selected at random, Omer, Gal, and Lior. We know that all three passed the first obstacle.

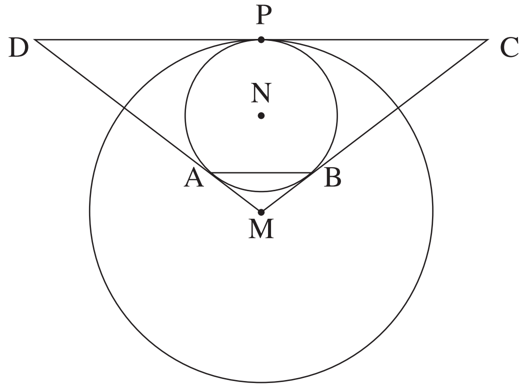
(1) Calculate the probability of precisely two of them qualifying for the semi-finals.

(2) Calculate the probability that, of the three of them, only Omer and Gal will qualify for the semi-finals.

/Continued on page 4/

Part Two — Planar Geometry and Planar Trigonometry

4. Two circles are internally tangent to each other at Point P (see figure).
Points M and N are the centers of the circles,
and the radii of the circles are R_1 and R_2, respectively; $R_2 < R_1$.
A tangent that is common to both circles is drawn through Point P.
Two lines extend from Point M and are tangent at Points A and B to the circle whose center is N.
The two lines intersect the tangent that is common to both circles at Points C and D, as shown in the figure.



א. Prove that $AB \perp MN$.

ב. Prove that $AB \parallel DC$.

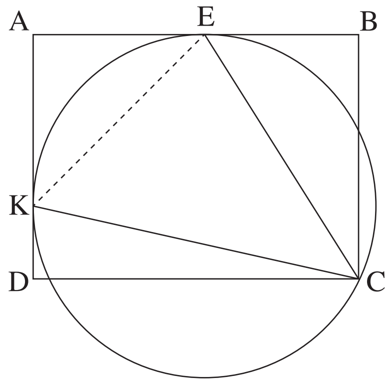
ג. Prove that $NB \cdot MC = MN \cdot \frac{DC}{2}$.

Given: $MN = 8$, $\frac{R_1}{R_2} = \frac{7}{3}$.

ד. **(1)** Find R_1 and R_2.

(2) Find DC.

5. Quadrilateral ABCD is a rectangle. Two of its sides, AB and AD, are tangent, at Points E and K respectively, to a circle whose radius is R (see figure).
Point C is located on the circle.



א. Prove: $\sphericalangle KCE = 45^o$.

Given: $\sphericalangle KCD = \alpha$, $0^o < \alpha < 45^o$.

ב. **(1)** Express the angles of Triangle KCE in terms of α.

(2) Express the lengths of the sides of Triangle KCE in terms of R and α.

ג. Express the ratio $\frac{EB}{AE}$ in terms of α.

Given: $\frac{EB}{AE} = \frac{\sqrt{2}}{2}$.

ד. Calculate α.

/Continued on page 5/

Part Three—Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

6. Given: the function $f(x) = \frac{x^2}{\sqrt{x^2 - a^2}}$, $a > 0$ is a parameter.

Express your answers in terms of a , if necessary.

א. Find the domain [תחום ההגדרה] of the function $f(x)$.

ב. Prove that the function $f(x)$ is even.

ג. **(1)** Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes (if there are any).

(3) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

(4) Sketch a graph of function $f(x)$.

Given: function $(f(x))^2$ whose domain is identical to the domain of function $f(x)$.

ד. Find the coordinates of the extrema of function $(f(x))^2$, and determine what type of extrema they are.

Given: the function $g(x) = \frac{1}{(f(x))^2}$. The domain of function $g(x)$ is identical to the domain of function $f(x)$.

ה. Sketch a graph of the function $g(x)$, based on the previous items.

Insert: $a = 2$.

ו. Calculate the area of the region that is bounded by the graph of function $g(x)$, the x-axis , and the lines $x = 3$ and $x = 4$.

/Continued on page 6/

7. Given: the function $f(x) = \dfrac{\cos^2(x)}{\sin(x)} + 3$.

Answer the items below for the interval $0 \le x \le 2\pi$.

א. **(1)** Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

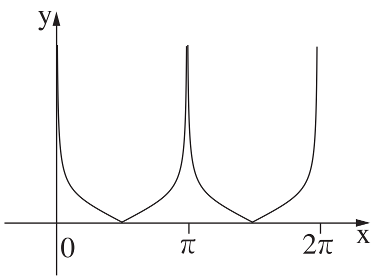
(3) Find the intervals on which the function $f(x)$ is increasing and decreasing.

(4) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

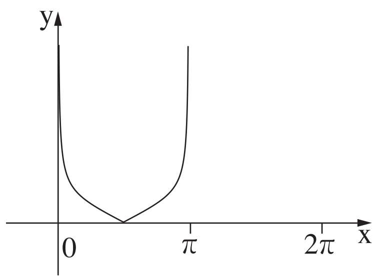
ב. Sketch a graph of function $f(x)$.

Two functions are given: $k(x) = f(x) - 3$, $g(x) = \sqrt{f(x) - 3}$.

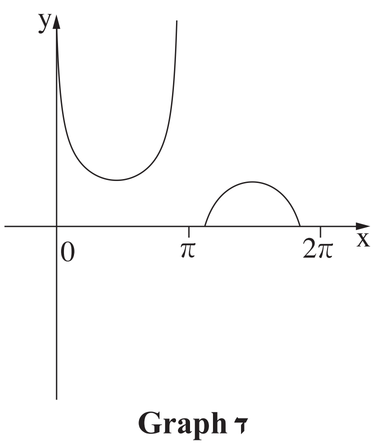
ג. One of the graphs א–ד below describes function $k(x)$, and one describes function $g(x)$.
Determine which of the graphs describes each of these functions, and explain your answers.



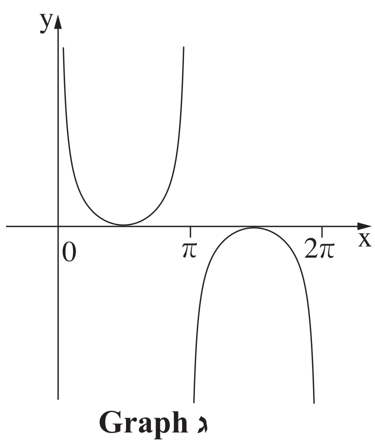
Graph ב



Graph א



Graph ד

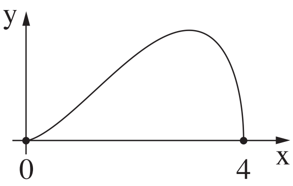


Graph ג

/Continued on page 7/

8. The figure below shows the function $f(x) = \sqrt{a \cdot x^4 + b \cdot x^3}$.

Given: The domain [תחום ההגדרה] of function $f(x)$ is $0 \le x \le 4$.



א. **(1)** Prove that $b = -4 \cdot a$.

(2) Below are two propositions [טענות] I-II. Only one of them is correct. Determine which is the correct proposition, and explain your answer.

I. $a > 0 , b < 0$

II. $a < 0 , b > 0$

Point P is located on the graph of function $(f(x))^2$, which is also defined on the domain $0 \le x \le 4$. A line that is perpendicular to the x-axis is drawn from Point P .
M is the intersection point of this perpendicular line with the x-axis, and O is the origin.

ב. What is the x-coordinate of Point P for which the area of Triangle PMO is maximum? Explain your answer.

ג. For the x-coordinate you found in Item ב , express in terms of a the maximum area of Triangle PMO .

ד. If we know that the x-coordinate of Point P is on the interval on which function $(f(x))^2$ is <u>not</u> decreasing, what is the x-coordinate of Point P for which the area of Triangle PMO is maximum? Explain your answer.

Good Luck!

בהצלחה!