

Mathematics
4 Points – 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes

ב. Exam structure and breakdown of points:

This exam has two parts, with a total of five questions.

Part One — Sequences, Spatial Trigonometry

Part Two — Growth and Decay, Differential and
Integral Calculus of Trigonometric
Functions, Exponential and
Logarithmic Functions, and
Power Functions

Answer three questions of your choice

$$- 3 \times 33\frac{1}{3} = 100 \text{ points.}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page.
If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה
4 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון – סדרות, טריגונומטריה במרחב
פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות
טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות
חזקה

עליך לענות על שלוש שאלות לבחירתך

$$- 3 \times 33\frac{1}{3} = 100 \text{ נקודות.}$$

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון,
גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים,
בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer three of the questions 1-5.

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One — Sequences, Spatial Trigonometry

Sequences

1. Daniella bought a refrigerator in monthly installments. The first payment was 700 shekels, and after that, each payment was 30 shekels less than the payment preceding it.
 - א. For which number payment will Daniella pay 280 shekels?
 - ב. (1) Is it possible that Daniella's final payment will be payment number 29 ?
Explain your answer.
 - (2) What is the lowest possible payment in the sequence of payments that Daniella will make, and what number payment is it?

Naama bought a refrigerator at the same price as Daniella, but she paid 30 equal monthly payments of 280 shekels each.

- א. How many payments will Daniella make to pay off for her refrigerator?

Spatial Trigonometry

2. Given: a right pyramid $SABC$, whose base ABC is a right triangle, $\angle CAB = 90^\circ$ (see figure).

Given: $AB = 9$, $AC = 12$.

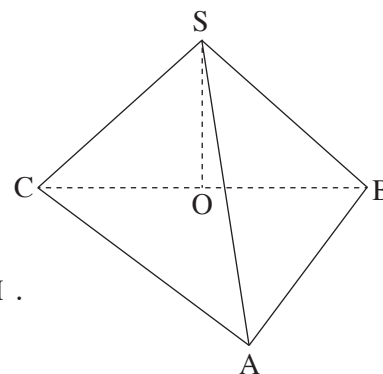
The angle between the side edge SB and the base ABC is 30° .

- א. Calculate the height of the pyramid, SO .
- ב. Calculate the volume of the pyramid.

Given: Point M is the midpoint of Side AB .

Point E is located on Segment OM , such that $OE = 2 \cdot EM$.

- א. Find the angle between SE and the base of the pyramid.
- ב. Calculate the area of Triangle SEM .

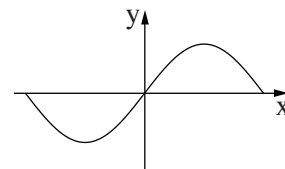


**Part Two — Growth and Decay, Differential and Integral Calculus of
Trigonometric Functions, Exponential and Logarithmic Functions,
and Power Functions**

3. The function $f(x)$, and its derivative function $f'(x)$ are defined on the domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

To the right is a figure showing the graph of the derivative function $f'(x)$.

We know that the graph of the derivative function $f'(x)$ intersects the x-axis at precisely three points: $(\frac{\pi}{2}, 0)$, $(0, 0)$, $(-\frac{\pi}{2}, 0)$.



- א. Find the intervals on which the function $f(x)$ is increasing and decreasing.
- ב. Find the x-coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and determine what type of extrema they are.

Given: $f(x) = (\sin x)^2 - \frac{1}{4}$.

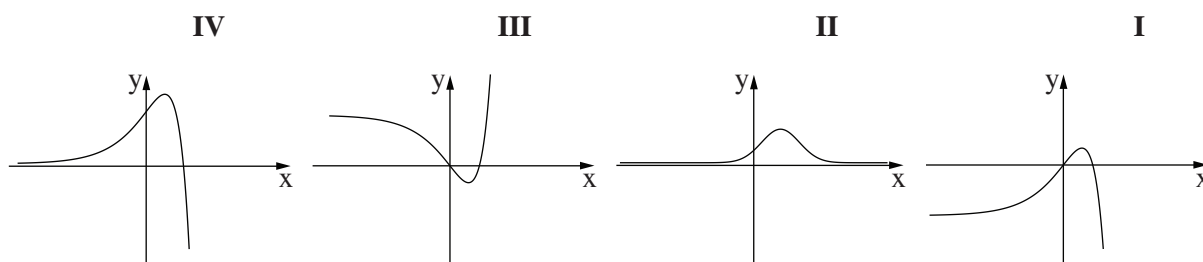
- ג. Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes.
- ד. Sketch a graph of the function $f(x)$.

A tangent to the graph of the function $f(x)$ was drawn at the function's intersection point with the x-axis, to the right of the origin. An additional tangent was drawn at the function's minimum.

- ה. Find the coordinates of the point at which these two tangents intersect.

Your answer should be correct to 2 decimal places.

4. Given: the function $f(x) = -e^{2x} + 4e^x - 3$, which is defined for any x .
- Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes.
 - Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$ and determine what type of extremum it is.
 - One of the graphs I-IV below describes the graph of the function $f(x)$. Determine which one it is and explain your answer.



Given: the function $g(x) = f(x) + b$. b is a parameter.

A tangent to the graph of the function $g(x)$ was drawn at its extremum.

- Find the equation of the tangent (express it in terms of b).
 - Find the area of the region bounded by the graph of the function $g(x)$, the tangent you found for Item א, and the y -axis.
5. Given: the function $f(x) = 3x \cdot \ln(ax)$. $a > 0$ is a parameter.

- Find the domain [תחום ההגדרה] of the function $f(x)$.

Given: The function $f(x)$ has an extremum [נקודת קיצון] at the point at which $x = \frac{1}{3e}$.

- Find a .

Insert $a = 3$, and answer Items ג-ו below.

- Find the coordinates of the point at which the graph of the function $f(x)$ intersects the x -axis.
- Find the y -coordinate of the extremum of the function $f(x)$ and determine what type of extremum it is.
- Sketch a graph of the function $f(x)$.
- Given: the function $g(x)$ whose domain is identical to the domain of the function $f(x)$.

The derivative function $g'(x)$ satisfies $g'(x) = -f(x)$.

Does the function $g(x)$ have an extremum?

If it does, find the x -coordinate of the extremum of $g(x)$, and determine what type of extremum it is. If it does not, explain.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך