The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2021, ***Moed Bet***
Exam Number: 035481
Appendix: Formula sheets for 4 points
English Translation (3)

מועד ב

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"א, 2021, **מועד ב**
מספר השאלון: 035481
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

Note: In this exam there are special instructions.
Answer the questions according to these instructions.

שים לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts with a total of eight questions.

Part One:	Algebra, Analytical Geometry, Probability
Part Two:	Planar Geometry and Planar Trigonometry
Part Three:	Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer four questions of your choice — $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/ foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet.
Write "טיוטה" at the top of each draft page.
If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון — אלגברה, גאומטריה אנליטית, הסתברות
פרק שני — גאומטריה וטריגונומטריה במישור
פרק שלישי — חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

עליך לענות על ארבע שאלות לבחירתך — $4 \times 25 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון,
גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים,
בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד.
רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-8 (each question — 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry, Probability

1. On a regular day, a train travels along a 300 km route at a constant speed.
One day the train traveled along the entire route at a speed that was 25% faster than its speed on a regular day, and therefore its travel time was half an hour less than on a regular day.

א. Find the speed of the train on a regular day and its travel time on a regular day.

On another day the train traveled for t minutes at the speed at which it travels on a regular day, but then had to reduce its speed by 10 km/h, and continued traveling at this lower speed until it reached the end of the route. On that day the train's travel time was 10 minutes longer that its travel time on a regular day.

ב. Find t .

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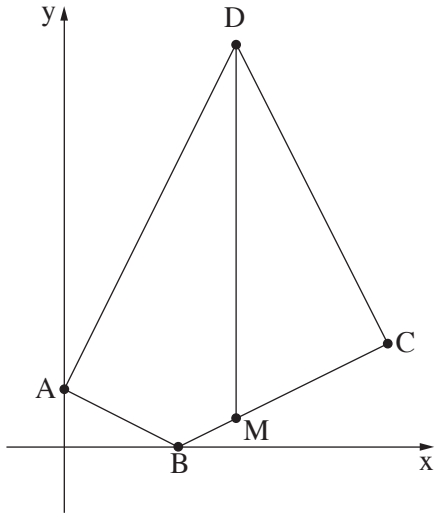
2. Given: Quadrilateral ABCD. Vertex A lies on the positive section of the y-axis and Vertex B lies on the x-axis.

Point M is located on Side BC, such that Line DM is parallel to the y-axis (see figure).



Given: The x-coordinate of Point M is 6 .

The equation of Side BC is: $y = \frac{1}{2}x - 2$.

א. Find the coordinates of Points B and M .

Given: $AB = 2 \cdot BM$.

ב. Find the coordinates of Point A .

Given: AD is perpendicular to AB .

ג. Find the coordinates of Point D .

Given: BC is perpendicular to CD .

ד. Find the equation of the circle that circumscribes Triangle MDC .

3. In a certain city a survey was conducted to investigate how many of the city's residents ride a bicycle.

The survey participants were divided into two groups: adults and young people.

Let x denote the probability of randomly choosing a young person from among the survey participants.

The survey found that:

80% of the young people ride a bicycle.

The number of young people who ride a bicycle is 4 times greater than the number of adults who do not ride a bicycle.

We know that the probability of randomly choosing a survey participant who does not ride a bicycle is 0.1 .

א. Find x .

A survey participant was randomly chosen.

ב. If we know that an adult was chosen, what is the probability that this adult rides a bicycle?

ג. What is the probability that the participant chosen is a young person or someone who rides a bicycle?

ד. We know that 3,850 adults who ride a bicycle participated in the survey.

How many people in total participated in the survey?

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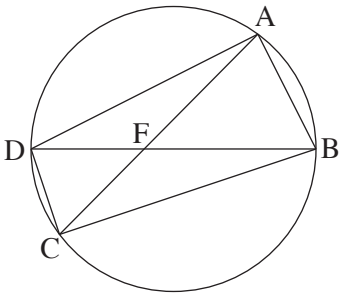
Part Two — Planar Geometry and Planar Trigonometry

4. Given: a circle. Chords AC and BD intersect at Point F (see figure).



א. Prove that $\triangle AFB \sim \triangle DFC$.

Given: $\sphericalangle DAB = \sphericalangle DCB$.

ב. Prove that BD is a diameter of the circle.

Given: $FC = \sqrt{18}$, $AF = \sqrt{32}$, $DF < BF$.

The radius of the circle is 5 .

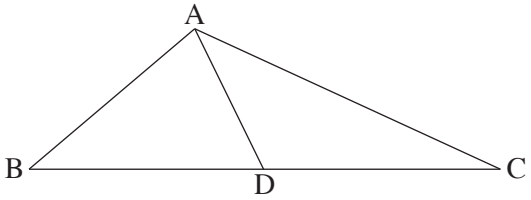
ג. Find the length of Segment BF .

Given: Point E is the midpoint of Segment AF , and Point G is the midpoint of Segment FB .

$DC = \sqrt{10}$.

ד. Find the length of Segment EG .

5. Given: Triangle ABC (see figure).



Given: $\frac{AC}{BC} = \frac{2}{3}$, $\sphericalangle BAC = 120^o$.

א. Calculate the size of the angle $\sphericalangle ABC$.

Given: BC = 12 .

AD is the median to Side BC of the triangle.

ב. Calculate the length of Segment AD .

Point F is the midpoint of Segment AD , and Point G is located on Side AB .

Given: The area of Triangle GAF equals 2 .

ג. Calculate the length of Segment AG .

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Part Three — Differential and Integral Calculus of Polynomials, of Rational Functions, and of Root Functions

6. Given: the function $f(x)=\frac{a}{6x^2-x^3}$. $a>0$ is a parameter.

א. **(1)** Find the domain [תחום ההגדרה] of the function $f(x)$.

(2) Write out the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes.

ב. Find the x-coordinate of the extremum [נקודת הקיצון] of the function $f(x)$, and determine what type of extremum it is.

ג. Find the intervals on which the function $f(x)$ is increasing and decreasing.

Given: The y-coordinate of the extremum of the function is $\frac{1}{4}$.

ד. Find a .

ה. **(1)** Sketch a graph of the function $f(x)$.

(2) Sketch a graph of the derivative function $f'(x)$.

ו. Calculate the area of the region bounded by the graph of the derivative function $f'(x)$, the x-axis, and the line $x=2$.

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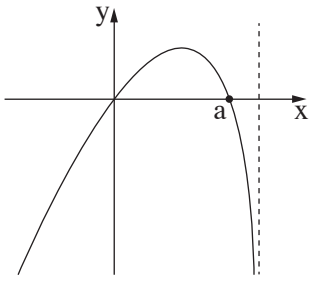
7. f(x) is a function whose derivative function $f'(x)$ is described by the graph in the figure on the right.



The graph intersects the x-axis only at the origin and at the point at which $x = a$.

a is a positive parameter.

א. Find the x-coordinates of the internal extrema [נקודות הקיצון] of the function f(x), and determine what type of extrema they are, based on the graph (if necessary, express them in terms of a). Explain your answer.

Given: $f(x) = x^2 \cdot \sqrt{5-x}$.

ב. Find the domain [תחום ההגדרה] of the function f(x).

ג. Find a.

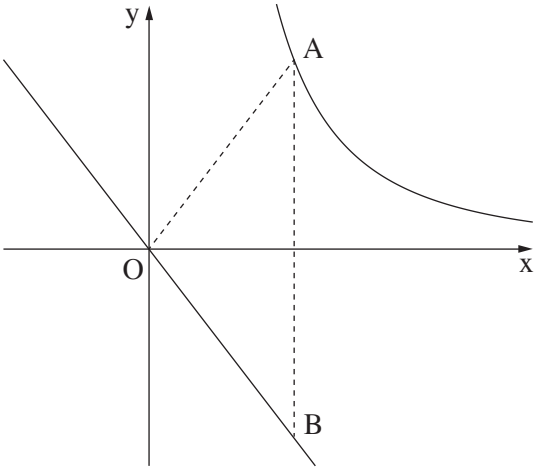
ד. Find the coordinates of the intersection points of the graph of function f(x) with the x-axis.

ה. Sketch a graph of function f(x).

Given: the function $g(x) = -3f(x)$.

ו. Find the equations of the tangents to the graph of the function g(x) that have a slope of 0.

8. The figure to the right describes the graph of the function $f(x) = \frac{9}{x^2}$, which is defined for any $x > 0$.
Also shown is the line $y = -\frac{4}{3}x$.



Point A is located on the graph of the function f(x) in the first quadrant.

From Point A, a line was drawn parallel to the y-axis. This line intersects the line $y = -\frac{4}{3}x$ at Point B.

א. Find the coordinates of Point A for which the area of Triangle AOB is minimal (O is the origin).

ב. Is there a Point A for which the area of Triangle AOB is 4 ? Explain your answer.

Good Luck!

בהצלחה!