

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

This exam consists of two parts, with a total of five questions.

Part One: Analytical Geometry, Vectors,

Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions,

Exponential and Logarithmic Functions

Answer three questions of your choice

$$-3 \times 33\frac{1}{3} = 100 \text{ points.}$$

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.

Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or

lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you use any draft paper outside the answer booklet your exam may be disqualified.

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

עליך לענות על שלוש שאלות לבחירתך – $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול

לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה מסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Good Luck!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer three of the questions 1-5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Given: Point K is located on the parabola $y^2 = 4px$ ($p > 0$).

The y-coordinate of Point K is 12.

The distance between Point K and the focus of the parabola is 20.

- א. Find p (find both possibilities).

Let p_1 and p_2 denote the values of p that you found in Item א. $p_1 < p_2$.

A line of the form $y = mx$ ($m \neq 0$) intersects the parabola $y^2 = 4p_1x$ at the origin and at another point, A, and also intersects the parabola $y^2 = 4p_2x$ at the origin and at another point, B.

Substitute the values p_1 and p_2 that you found, and answer Items ב and ג.

- ב. Express the coordinates of Point A and the coordinates of Point B in terms of m .

For every line $y = mx$ ($m \neq 0$), let M represent the midpoint of the segment AB that is formed in the manner described.

- ג. Find the equation (without using m) of the locus on which these points M are located.

2. Consider the given cuboid $ABCD A'B'C'D'$.

Point K is located on the edge CC' .

Point E is the midpoint of the edge $A'D'$ (see figure).

Let: $\overrightarrow{CK} = t \cdot \overrightarrow{CC'}$ ($t > 0$ is a scalar); $\overrightarrow{AB} = \underline{u}$; $\overrightarrow{AD} = \underline{v}$; $\overrightarrow{AA'} = \underline{w}$.

Given: $|\underline{u}| = 3\sqrt{2}$; $|\underline{v}| = 6$; $|\underline{w}| = 6\sqrt{2}$,

$$\angle EKB = 90^\circ$$

א. Find t .

Let π be the plane $CDA'B'$.

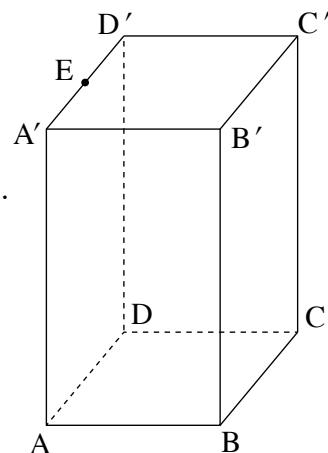
ב. (1) Prove that the line BK is perpendicular to the plane π .

(2) Explain why the line EK is parallel to the plane π .

Given: $K(4, 5, -1)$; $B(-1, 0, 1)$,

$$\underline{w} = (2, 2, -8).$$

ג. Find the equation of the plane π .



3. א. Solve the equation: $z^2 - (1 + i)z + 2i + 2 = 0$.

One of the solutions to the equation you solved is located in the fourth quadrant and is represented by Point A on the Gauss plane.

The second solution is represented by Point B on the Gauss plane.

A circle whose center is the origin, O , passes through Point B . The line AO intersects the circle at Points C and D .

A regular polygon with n sides is inscribed in the circle.

Given: Points B , C , D are vertices of this polygon.

ב. What is the smallest possible n ? Explain your answer.

ג. For the value of n that you found in Item ב:

(1) List the complex numbers that correspond to the polygon's vertices.

(2) Write an equation whose solutions are all the complex numbers that correspond to the polygon's vertices.

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. The function $g(x)$ is defined and differentiable [גזירה] for any x . Its graph intersects the x -axis only at the origin.

The extrema of the function $g(x)$ are $(1, 1)$ and $(-1, -1)$, only.

The derivative of the function $g(x)$ vanishes [מתאפסת] for $x = 1$ and for $x = -1$ only.

The x -axis is a horizontal asymptote of the function $g(x)$ when x approaches infinity and when x approaches minus infinity.

- א. (1) Sketch a possible graph of the function $g(x)$.
(2) List the positive and the negative intervals of $g'(x)$ (the derivative function of $g(x)$).

Consider the given function $f(x) = e^{g(x)} - g(x)$.

- ב. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
(2) Find the horizontal asymptote of the function $f(x)$.
(3) Find the coordinates of the extrema of the function $f(x)$ and determine what type of extrema they are.
(4) Sketch a possible graph of the function $f(x)$.

5. Consider the given family of functions $f(x) = ax - \ln\left(\frac{x}{a}\right)$, $a \neq 0$ is a parameter.

Answer Items א to ג for $a > 0$ and for $a < 0$.

- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
ב. Express in terms of a the coordinates of the extremum of the function $f(x)$, and determine what type of extremum it is.

Given: The function $f(x)$ intersects the x -axis at two different points.

- ג. Sketch a graph of the function $f(x)$.

The given function $g(x) = f'(x)$ (the derivative of the function $f(x)$) is defined on the same domain as the function $f(x)$.

Given: $a > 0$.

- ד. Find the area of the region bounded by the graph of the function $g(x)$, the line $x = \frac{2}{a}$, and the x -axis, and prove that this area is not dependent on a .

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל.
אין להעתיק או לפרסם
אלא ברשות משרד החינוך.