The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: **Winter - late,** 2021
Exam Number: 035481
Appendix: Formula sheets for 4 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: **חורף מאוחר**, תשפ"א, 2021
מספר השאלון: 035481
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

מועד חורף מאוחר

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam consists of three parts, a total of eight questions.

Part One – Algebra, Analytical Geometry, Probability
Part Two – Planar Geometry and Planar Trigonometry
Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer five questions of your choice – 5 × 20 = 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון — אלגברה, גאומטריה אנליטית, הסתברות
פרק שני — גאומטריה וטריגונומטריה במישור
פרק שלישי — חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

עליך לענות על חמש שאלות לבחירתך — 5×20 = 100 נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Answer five of the questions 1-8 (each question — 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry, Probability

1. Uri and Danny set off from two different points at the same time, and walked toward each other along the same straight route.

Uri's walking speed was 40% faster than Danny's walking speed.

Uri and Danny met one and a quarter hours after setting off.

Let x be Danny's walking speed.

א. Express the length of the route in terms of x.

40 minutes after setting off, the distance between Uri and Danny was 4.9 km.

ב. Find Danny's walking speed and Uri's walking speed.

ג. What was the distance between Uri and Danny one hour after setting off?

2. The line $y=-2$ is tangent at Point A to a circle whose center is M (see figure).

The center of the circle is located in the first quadrant.

The radius of the circle is 5.

א. Find the y-coordinate of the center of the circle.

Given: The circle passes through the point $C(2,0)$.

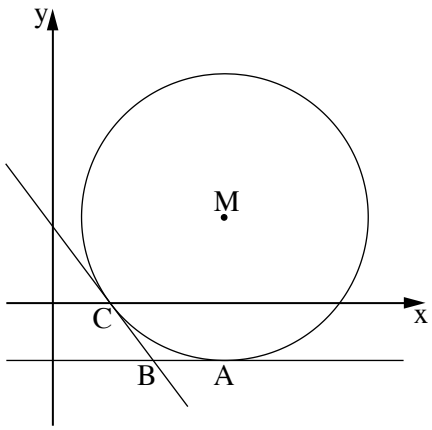
ב. Find the equation of the circle.

The tangent to the circle at Point C intersects the line $y=-2$ at Point B (see figure).

ג. Find the area of the quadrilateral MABC.

D is a point on the circle such that AD is a diameter of the circle.

ד. Find the equation of the tangent to the circle at Point D.



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3. In the Oz Republic the number of residents who have a driver's license is 3 times greater than the number of residents who do not have a driver's license.

א. What is the probability that a resident of the Oz Republic has a driver's license?

Given: 60% of the residents of the Oz Republic are adults, and the rest are young people. 80% of the adult residents of the Oz Republic have a driver's license.

ב. What is the probability that a resident of the Oz Republic is a young person who does not have a driver's license?

ג. A resident of the Oz Republic is chosen at random, and we know that this resident is a young person. What is the probability that this resident has a driver's license?

ד. 4 residents of the Oz Republic are chosen at random. What is the probability that exactly 2 of them are young people who have a driver's license?

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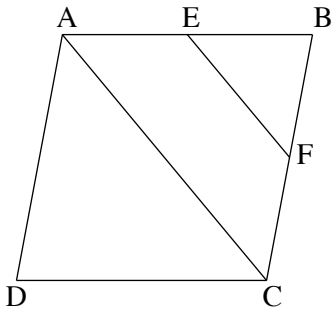
Part Two — Planar Geometry and Planar Trigonometry

4. The figure on the right describes a rhombus, ABCD .

Points E and F are the midpoints of Sides AB and BC , respectively.



א. Prove that $EF \parallel AC$.

ב. **(1)** Prove that $\Delta EBF \sim \Delta ABC$.

(2) Find the ratio of the area of Triangle EBF to the area of Rhombus ABCD .

ג. Prove that $BD \perp EF$.

Given: The perimeter of the rhombus is 32 ;

$EF = 2\sqrt{7}$.

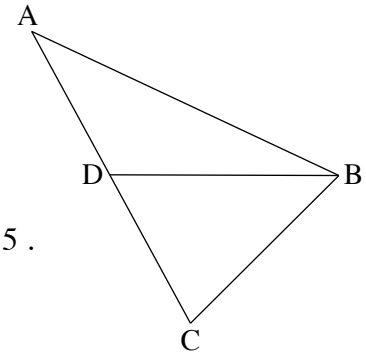
M is the point of intersection of BD and EF .

ד. **(1)** Find BM .

(2) Find MD .

5. The figure on the right describes an acute-angle triangle ABC .

BD is the median to Side AC .



Given: $DB = a$, $AB = 1.5a$, $\sphericalangle ABD = 28^{o}$.

א. Express the length of the section AD in terms of a .

Given: The radius of the circle circumscribing Triangle ABD is 5 .

ב. Find a .

ג. Calculate the area of Triangle ABC .

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Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Given: the function $f(x)=\frac{16}{x^2}-x^2$.

א. **(1)** Find the domain [תחום ההגדרה] of the function $f(x)$.

(2) Find the asymptotes of the function $f(x)$ that are perpendicular to the axes (if there are any).

ב. Find the intervals on which the function $f(x)$ is increasing and on which it is decreasing.

ג. Find the coordinates of the points at which the graph of the function $f(x)$ intersects the x-axis.

ד. Sketch a graph of the function $f(x)$.

ה. Find the area of the region bounded by the graph of the function $f(x)$, the x-axis, and the vertical line $x = 1$.

7. Given: the function $f(x)=ax\cdot\sqrt{12-x}$. $a>0$ is a parameter.

א. Find the domain [תחום ההגדרה] of the function $f(x)$.

ב. Find the coordinates of the points at which the graph of the function $f(x)$ intersects the x-axis.

ג. Find the coordinates of all the extrema [נקודות הקיצון] of the function $f(x)$, and determine what type of extrema they are (if necessary, express in terms of a).

ד. Sketch a graph of the function $f(x)$.

The function $g(x)=f(x)-32$ is tangent to the x-axis.

ה. **(1)** Sketch a graph of the function $g(x)$.

(2) Find a .

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8. In the figure on the right, ABCD is a rectangle whose area is 25 .

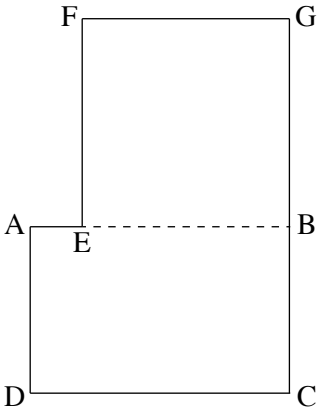
Point E is located on Side AB ,

and Point G is located on the extension of Side CB , as shown in the figure.

Quadrilateral EFGB is a square.

Given: $AD < DC$;

the side of the square is 25% longer than AD.



Let $AD = x$.

א. Express DC and AE in terms of x .

ב. Find the value of x for which the perimeter of the polygon AEFGCD that is formed is minimal.

Good Luck!

בהצלחה!