The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2021
Exam Number: 035482
Appendix: Formula sheets for 4 points
English Translation (3)

Mathematics
4 Points — 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes.

ב. Exam structure and breakdown of points:
This exam has two parts.

Part One — Sequences, Spatial Trigonometry
Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

Answer three questions of your choice

— $3 \times 33\frac{1}{3}$ — 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator.
Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you use any draft paper outside the answer booklet your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשפ"א, 2021
מספר השאלון: 035482
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה
4 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים.
פרק ראשון — סדרות, טריגונומטריה במרחב
פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה

עליך לענות על שלוש שאלות לבחירתך — $3 \times 33\frac{1}{3}$ — 100 נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Answer three of the questions 1-5.

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One — Sequences, Spatial Trigonometry

Sequences

1. Given: two sequences: $a_n = 4n + 1$, $b_n = 9 - 2n$.

א. **(1)** Find a_1 and b_1 .

(2) Prove that both sequences are arithmetic sequences, and find the common difference of each one.

The sum of the first k terms in sequence a_n is 860 .

ב. **(1)** Find k .

(2) Find the sum of the first k terms in the sequence b_n .

Given: a new sequence which satisfies $c_n = a_n - b_n$ for any n .

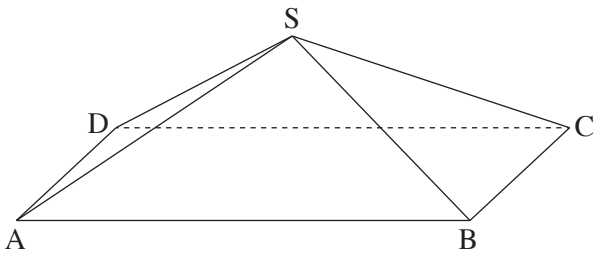
ג. Is c_n an arithmetic sequence? Prove your answer.

ד. What is the sum of the first 20 terms in sequence c_n ? Explain your answer.

Spatial Trigonometry

2. SABCD is a right pyramid with a rectangular base.

Given: $BC = 2$, $AB = 6$, $SC = 4$.



א. Calculate the length of the diagonal of the pyramid's base.

ב. **(1)** Find the angle between a lateral edge of the pyramid and the pyramid's base.

(2) Find the area of the triangle ASC .

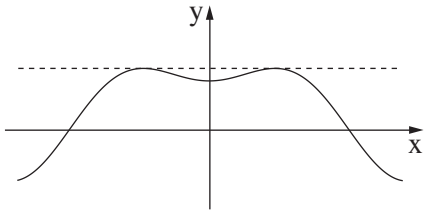
ג. **(1)** Find the apex angles [זווית הראש] of all the lateral faces of the pyramid.

(2) Calculate the lateral surface area of the pyramid.

/Continued on page 3/

Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

3. The figure on the right describes the graph of the function $f(x)$ which is defined over the interval $-\pi \le x \le \pi$.



Given: $f(x) = -\frac{1}{2}\cos(2x) + \cos x + c$. c is a parameter.

א. Find the x-coordinates of all the extrema [נקודות הקיצון] of the function $f(x)$. Use the graph of the function to determine what type of extrema they are.

Given: The line $y = 1.25$ is tangent to the graph of the function $f(x)$ at the maxima of the function (see figure).

ב. **(1)** Find c.

(2) List the coordinates of the minima of the function $f(x)$.

Given: the function $g(x) = f(x) + b$ (b is a parameter).

ג. Find the value of b for which the function $g(x)$ is tangent to the line $y = 0.25$ (find all three options).

4. Given: the function $f(x) = \frac{a \cdot e^x}{e^x - a}$, $a > 0$ is a parameter.

א. **(1)** Express the domain [תחום ההגדרה] of the function $f(x)$ and the equation of the vertical asymptote of the function $f(x)$ in terms of a.

(2) Express the coordinates of the intersection points of the graph of the function $f(x)$ with the axes in terms of a (if there are any such points).

ב. Express in terms of a the intervals on which the function $f(x)$ is decreasing.

Given: The point of intersection of the graph of the function $f(x)$ with the y-axis is $(0, -2)$.

ג. Find a.

The function $f(x)$ has two horizontal asymptotes: $y = 2$ for $x \to \infty$ and $y = 0$ for $x \to -\infty$.

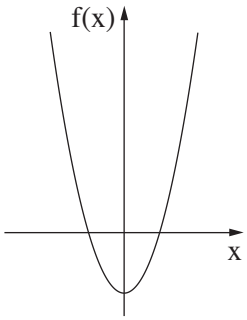
ד. Sketch a graph of the function $f(x)$.

ה. Given: the function $g(x) = |f(x)|$.

What are the coordinates of the intersection points of the graph of the function $g(x)$ with the y-axis?

/Continued on page 4 /

5. The graph in the figure to the right describes the function $f(x)=x^2-4$ which is defined for any x.



א. **(1)** Find the coordinates of the intersection points of the graph of the function $f(x)$ with the axes.

(2) What are the positive and negative intervals of the function $f(x)$?

Given: the function $g(x)=\ell n\big(f(x)\big)$.

ב. **(1)** What is the domain [תחום ההגדרה] of the function $g(x)$?

(2) Find the equations of the vertical asymptotes of the function $g(x)$.

(3) Find the coordinates of the points of intersection of the graph of the function $g(x)$ with the x-axis. You may leave a root sign in your answer.

(4) What are the intervals on which the function $g(x)$ is increasing and the intervals on which it is decreasing?

(5) Sketch a graph of the function $g(x)$.

Good Luck!