

Mathematics

4 Points — 1st Exam

Instructions for examinees

- א. Duration of the exam: Three and a half hours
- ב. Exam structure and breakdown of points:
This exam consists of three parts, a total of eight questions.
Part One — Algebra, Analytical Geometry, Probability
Part Two — Planar Geometry and Planar Trigonometry
Part Three — Differential and Integral Calculus of
Polynomials, Rational Functions, and
Root Functions
Answer five questions of your choice — $5 \times 20 = 100$ points

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

- א. משך הבחינה: שלוש שעות וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון — אלגברה, גאומטריה אנליטית, הסתברות
פרק שני — גאומטריה וטריגונומטריה במישור
פרק שלישי — חשבון דיפרנציאלי ואינטגרלי של
פולינומים, של פונקציות רציונליות ושל
פונקציות שורש
עליך לענות על חמש שאלות לבחירתך — $5 \times 20 = 100$ נקודות.

- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

- ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer five of the questions 1-8 (each question — 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry, Probability

1. The distance between City א and City ב is 54 km.

Two cyclists, Dana and Hila, set out toward each other at the same time. Dana set out from City א to City ב and Hila set out from City ב to City א. Each cycled at a constant speed.

One hour after Dana and Hila set out, they met. After they met, Hila cycled on to City א at the same speed as before whereas Dana stopped for an hour and a half and then cycled on to City ב at the same speed as before.

Dana arrived at City ב at exactly the same time as Hila arrived at City א.

א. Find at what speed Dana cycled and at what speed Hila cycled.

ב. What is the ratio between the distance that Dana covered from the moment they met until she arrived in City ב, and the distance that Hila covered from the moment they met until she arrived in City א?

2. Point A(1, 8) is located on a circle whose center is M(3, 5).

א. Find the equation of the circle.

A tangent to the circle was drawn through Point A

(see figure).

ב. Find the equation of the tangent.

Point B is located on the circle.

Its y-coordinate is 2 and its x-coordinate is greater than 3.

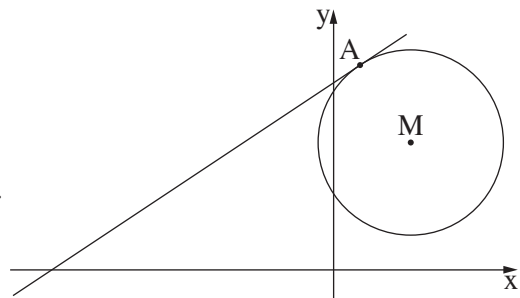
ג. (1) Find the x-coordinate of Point B.

(2) Is AB a diameter of the circle? Explain.

A line is drawn from Point B, parallel to the x-axis and intersecting the tangent at Point C.

Point E is the center of the circle that circumscribes Triangle CAB.

ד. Calculate the area of Triangle EAC.



3. Menachem took a driving test in order to get his driving license.

The probability of Menachem failing a driving test is fixed and is 4 times greater than the probability that he passes it.

- א. What is the probability that Menachem passes a driving test?

Every time that Menachem does not pass the driving test, he takes another one until he passes the driving test.

We know that Menachem got his driving license.

- ב. (1) What is the probability that Menachem took at most two tests?
(2) What is the probability that Menachem took a second test if we know that he took at most two tests?

Part Two — Planar Geometry and Planar Trigonometry

4. BC is a diameter of a circle whose center is O.

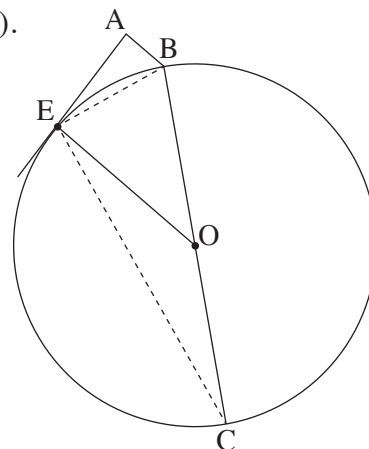
A tangent to the circle is drawn through Point E, which is located on the circle.

Point A is located on the tangent such that $AB \parallel EO$ (see figure).

- א. Prove that $\angle EAB = 90^\circ$.
- ב. Prove that $\triangle EAB \sim \triangle CEB$.
- ג. Prove that $EB^2 = AB \cdot CB$.

Given: $\frac{CB}{EB} = 3$.

- ד. Calculate the ratio between the area of Triangle EBO and the area of Triangle EAB $\left(\frac{S_{\triangle EBO}}{S_{\triangle EAB}} \right)$.



5. Point D is located on Side AB of Triangle ABC such that $AD = \frac{1}{3}AC$ (see figure).

Given: $\angle ACD = 15^\circ$,

Angle ADC is an acute angle.

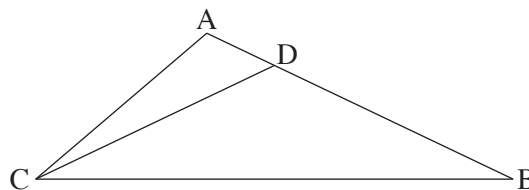
- א. Calculate the size of Angle ADC.

Given: CDB is an isosceles triangle ($CD = DB$) whose area is 40.

- ב. Calculate the length of Segment AD.

Point P is the midpoint of Segment CD.

- ג. Calculate the length of Segment PB.



Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Given: the function $f(x) = \frac{bx^2}{x^2 - 4x + 3}$. b is a parameter.

The function $f(x)$ has a horizontal asymptote whose equation is $y = 2$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.

- (2) What is the value of parameter b ?

Substitute the value you found for b and answer Items ג-ב.

- ב. (1) Find the equations of the vertical asymptotes of the function $f(x)$.

- (2) Find the coordinates of the intersection points of the graph of the function $f(x)$ with the axes (if there are any).

- (3) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$ and determine what type of extrema they are.

- ג. Sketch a graph of the function $f(x)$.

7. The figure on the right describes the graph of the function

$$f(x) = \frac{6}{x} \text{ which is defined for any } x \neq 0.$$

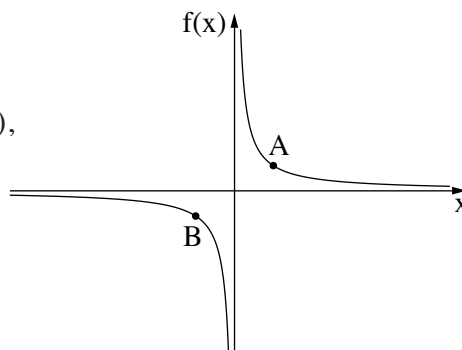
Points A and B are located on the graph of the function $f(x)$, as shown in the figure.

$$x_A = t \text{ and } x_B = -t$$

א. Express AB^2 in terms of t .

ב. (1) For what value of t is AB^2 minimal?

(2) Use Sub-item ב (1) to determine for what value of t the length of Segment AB is minimal. Explain.

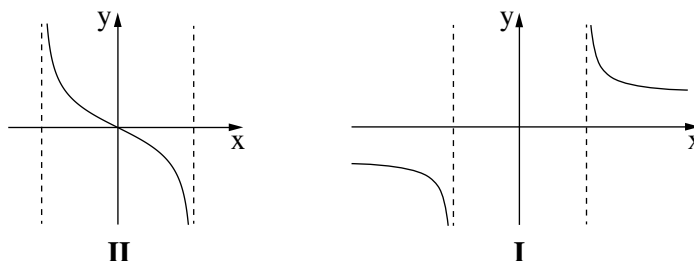


8. The figures below show two graphs, I and II.

Each graph describes the derivative function of another function.

Graph I never intersect the axes; Graph II intersects the axes only at Point $(0, 0)$.

Each of the two graphs has vertical asymptotes whose equations are $x = \pm 2$.



א. Using Graphs I and II, find the intervals of increase and of decrease of each of the functions whose derivatives are described by each of the graphs.

Given: two functions $f(x) = \sqrt{4 - x^2}$, $g(x) = \sqrt{x^2 - 4}$. Each of the graphs I and II shows the derivative function of one of these functions.

- ב. (1) Find the domain [תחום ההגדרה] of each of the functions $f(x)$ and $g(x)$.
(2) Match each of the graphs I and II with the corresponding derivative function, $(f'(x) \text{ or } g'(x))$. Explain.

Answer Item א for each of the functions $f(x)$ and $g(x)$.

- ג. (1) Find the coordinates of the intersection points of the graph of the function with the axes.
(2) Sketch a graph of the function.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך