

**The State of Israel
Ministry of Education**

Exam Type: *Bagrut*

Exam Date: Summer 2020, *Moed Bet*

Exam Number: 035481

Appendix: Formula sheets for 4 points

English Translation (3)

מדינת ישראל

משרד החינוך

סוג הבחינה: בגרות

מועד הבחינה: קיץ תש"ף, 2020, מועד ב'

מספר השאלון: 035481

נספח: דפי נוסחאות ל-4 יחידות לימוד

תרגום לאנגלית (3)

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry, Probability

— 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus of

Polynomials, of Rational Functions, and of
Root Functions

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet.

Write "טיטה" at the top of each draft page.

If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית,

הסתברות — 2×20 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

— 1×20 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

של פולינומים, של פונקציות רציונליות

ושל פונקציות שורש

— 2×20 — 40 נקודות

סך הכול — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד.

רשום "טיטה" בראש כל עמוד המשמש טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Yael and Alon ran a relay race [מרוץ שליחים] of 15 kilometers in total.

At the beginning of the race, Yael was standing at the starting line and Alon was standing on the racecourse 5 kilometers away from her.

Yael ran at a constant speed of V km/h until she reached Alon.

As soon as Yael reached Alon, he set off and ran to the end of the racecourse and Yael returned to the starting line.

Alon ran at a constant speed that was faster than Yael's initial speed by 2 km/h.

Yael returned to the starting line at a constant speed of $\frac{5}{6}V$ km/h.

Alon arrived at the end of the racecourse 15 minutes after Yael arrived back at the starting line.

א. (1) Express in terms of V the time that it took Yael to run from the starting line to Alon.

(2) Express in terms of V the time it took Yael to return (the time from the moment Yael reached Alon until she arrived back at the starting line).

(3) Find V (find both possibilities).

We know that the entire race (from the moment Yael started running until Alon arrived at the end of the racecourse) took less than two hours.

ב. Which of the two possibilities you found in sub-item א(3) is V ? Explain.

2. The following figure describes a circle whose center, M , is in the second quadrant.

The circle passes through the origin, O , and its radius is 5.

Given: The center of the circle, M , is located on the line $y = 3$.

- א. Find the equation of the circle.

The circle intersects the x -axis at another point, A .

- ב. Find the coordinates of Point A .

A tangent to the circle is drawn through Point A .

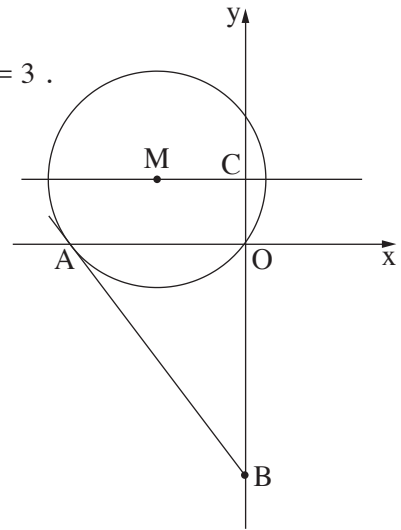
This tangent intersects the y -axis at Point B .

- ג. Find the coordinates of Point B .

The line $y = 3$ intersects the y -axis at Point C .

A line is drawn from Point M parallel to the y -axis and intersecting Line AB at Point D .

- ד. Calculate the area of the trapezoid $MCBD$.



3. In a large field of flowers there are flowers of three colors.

$\frac{1}{3}$ of the flowers are white, $\frac{1}{4}$ of the flowers are yellow, and the rest are purple.

Yosi and Vered picked flowers in this field.

Yosi picked two random flowers.

- א. What is the probability that both flowers that Yosi picked were the same color?
 ב. We know that Yosi picked two flowers of the same color.
 What is the probability that the two flowers were yellow?

Vered is making bouquets from the random flowers she is picking in the field.

Every bouquet has exactly 5 flowers.

- ג. (1) What is the probability that one bouquet that Vered has made has at least one purple flower?
 (2) Vered made 3 bouquets. What is the probability that each of the bouquets she made contained at least one purple flower?

Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. The figure to the right describes Parallelogram ABCD.

G is a point on Diagonal AC

in the parallelogram and

F is a point on Side AD.

Given: $\angle FGA = \angle ABC$.

א. (1) Prove that: $\triangle FGA \sim \triangle ABC$

(2) Prove that: $AF \cdot DC = FG \cdot AC$

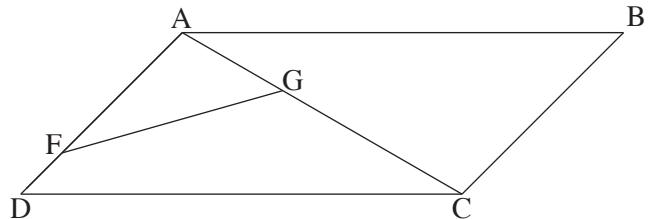
Given: The area of Triangle ABC is 20, and the area of Triangle FGA is 5.

ב. Calculate the ratio $\frac{AF}{AC}$.

Given: $FG \parallel DB$,

The parallelogram diagonals intersect at Point H.

ג. Prove that: $\triangle ABC \sim \triangle BHC$.



5. The triangle ABC is inscribed in a circle (see figure).

Given: $AC = 7$, $BC = 3$, $AB = 5$.

א. (1) Find the size of the angle ACB.

(2) Find the radius of the circle circumscribing Triangle ABC.

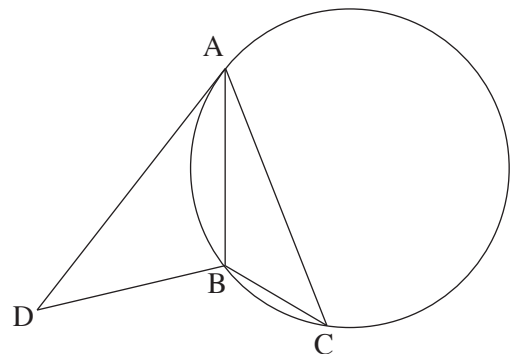
A tangent to the circle is drawn at Point A.

Point D is located on the tangent such that

the area of Triangle DBA is 12.

ב. Find the length of Side AD.

ג. Find the ratio of the radius of the circle circumscribing Triangle DBA to the radius of the triangle circumscribing Triangle ABC.



Continued on page 5 ►

Part Three — Differential and Integral Calculus of Polynomials, of Rational Functions, and of Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = \frac{3x}{x^2 - 1} + 2$.

- א. (1) What is the domain of the function $f(x)$?
- (2) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes.
- (3) Find the coordinates of the intersection points of the graph of the function $f(x)$ with the axes.
- (4) Find the intervals on which the function $f(x)$ increases and the intervals on which it decreases (if there are any).
- (5) Sketch a graph of the function $f(x)$.

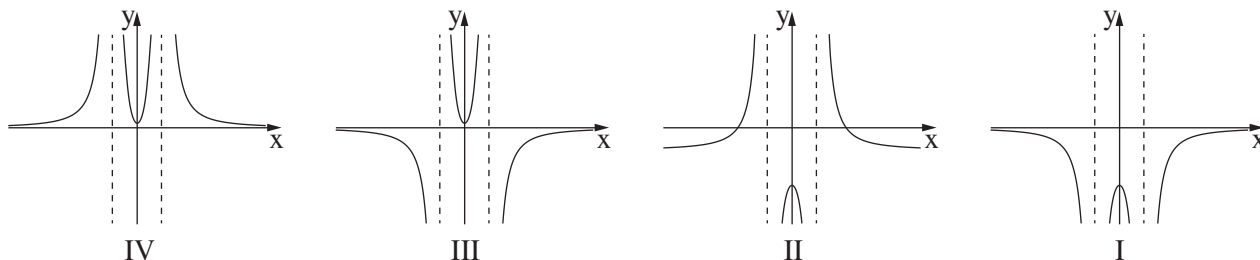
At the end of this question you will find sketches of four graphs (I-IV). One of them is a graph of the derivative function $f'(x)$.

א. Which of the graphs I-IV is a graph of the derivative function $f'(x)$? Explain.

א. $a > 3$ is a parameter.

The area bounded by the graph of the derivative function $f'(x)$, the lines $x = 3$ and $x = a$, and the x -axis equals 0.5.

Find the value of a .



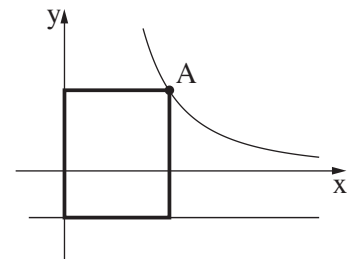
7. Given: the function $f(x) = -2x \cdot \sqrt{-x^2 + 8}$

- א. (1) What is the domain of the function $f(x)$?
- (2) Find the coordinates of the intersection point of the graph of the function $f(x)$ with the y-axis.
- (3) Find the coordinates of the extrema of the function $f(x)$ and determine what type of extrema they are.
- ב. Sketch a graph of the function $f(x)$.
- ג. For which values of k does the line $y = k$ intersect the graph of the function $f(x)$ at exactly two points?
- ד. Sketch a graph of the function $-f(x)$ [minus $f(x)$].

8. The graph of the function $f(x) = \frac{4}{x^2}$ shown to the right is defined for every $x > 0$.

Point A is located on the graph of the function $f(x)$ in the first quadrant.

Perpendiculars were dropped from Point A to the y-axis and to the line $y = -1$, forming a rectangle with the y-axis and the line $y = -1$ as shown in the figure.



- א. For what coordinates of Point A is the area of the rectangle minimal?
- ב. Is there a Point A for which the area of the rectangle is 3? Explain.

Good Luck!

All rights reserved by the State of Israel.
 Copying or publication without the consent of the
 Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך