

Mathematics

3 Points — 3rd Exam

Instructions for examinees

א. Duration of the exam: Two hours

ב. Exam structure and breakdown of points:

This exam has six questions in the topic areas:

Algebra, Differential and Integral Calculus.

You must answer four questions — $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write in the answer booklet only. Write "טייטה" at the top of every page you use for rough work. If you write draft material outside the answer booklet, your exam may be disqualified.

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שעותיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות — $4 \times 25 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות

במחשבון שיש בו אפשרות לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון

עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה

ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד.

רשום "טייטה" בראש כל עמוד המשמש טייטה.

כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-6 (each question – 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. A greengrocer bought some tomatoes and peppers.

He paid 6 shekels per kilo for the tomatoes.

For each kilo of peppers he paid 10% more than he had paid for a kilo of tomatoes.

The greengrocer bought a total of 126 kilos of tomatoes and peppers, and paid 780 shekels for them.

א. (1) How many kilos of tomatoes did the greengrocer buy?

(2) How many shekels did he pay for the tomatoes he bought?

The greengrocer sold half of the tomatoes he had bought at a profit of 30% and sold the remaining half at a loss of 20%.

א. How much money did he receive from the sale of all the tomatoes?

ב. The greengrocer sold the peppers for 8 shekels per kilo.

What percentage profit did the greengrocer make from the sale of all the tomatoes and peppers?

Your answer should be correct to two decimal places.

2. The line $y = -2x + 5$ intersects the y-axis at Point A and the x-axis at Point B (see figure).

א. Find the coordinates of Points A and B.

Point O is the origin.

Point D is located on the y-axis, below the x-axis (see figure).

Given: $OD = 4 \cdot OB$

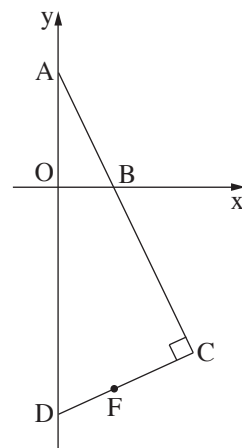
ב. Find the coordinates of Point D.

Point C is located on the extension of Segment AB, such that Line DC is perpendicular to Line AC.

- ג. (1) Find the equation of Line DC.
- (2) Find the coordinates of Point C.

Point F is located on Line DC; its y-coordinate is -8.75

- ד. (1) Prove that Line BF is parallel to the y-axis.
- (2) Write out the equation of Line BF.



3. The figure to the right shows a circle whose center is M and whose equation is $(x - 4)^2 + (y - 1)^2 = 25$. The circle intersects the y-axis at Points A and B, as shown in the figure.

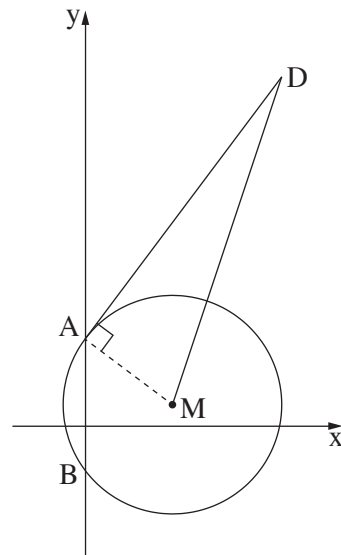
א. Find the coordinates of Points A and B.

- ב. (1) Find the slope of the tangent to the circle at Point A.
- (2) Find the equation of the tangent to the circle at Point A.

Point D is located on the tangent whose equation you found.

The equation of Line MD is $y = 3x - 11$.

- ג. Find the coordinates of Point D.
- ד. Calculate the area of Triangle AMD.



Differential and Integral Calculus

4. Given: the function $f(x) = \frac{1}{2} \cdot \frac{1}{x} - x$

- א. What is the domain of the function $f(x)$?
- ב. (1) Show that the function $f(x)$ has no extrema.
(2) Find the intervals on which the function $f(x)$ increases and decreases (if there are any).

A tangent to the graph of the function $f(x)$ was drawn at the point at which $x = 1$.

- ג. Find the equation of the tangent.
- ד. Find the point at which the tangent you found intersects the line $y = -2$.

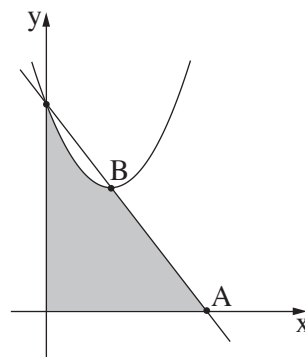
5. Given: the function $f(x) = x^2 - 4x + 10$ and the line whose equation is $y = -2x + 10$.

The line intersects the x-axis at Point A (see figure).

- א. Find the coordinates of Point A.

The line and the graph of the function $f(x)$ intersect on the y-axis and at an additional point, B.

- ב. Find the coordinates of Point B.
- ג. Calculate the area of the shaded region in the figure:
the region bounded by the graph of the function $f(x)$, the line, and the axes.



6. The figure to the right shows the graph of the function $f(x) = -x^3 + 2x$.

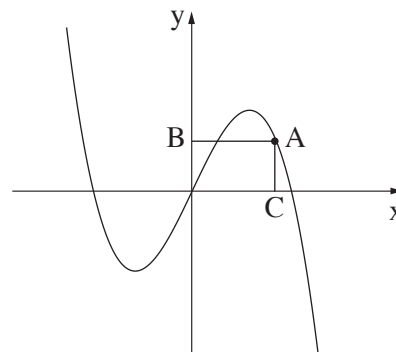
- א. Find the coordinates of the points at which the graph of the function $f(x)$ intersects the x -axis.

Your answer should be correct to two decimal places.

Point A is located on the graph of the function $f(x)$ in the first quadrant.

Perpendiculars were drawn from Point A to the axes, intersecting the axes at Points B and C, as shown in the figure.

- ב. Find the coordinates of Point A for which the sum $AB + AC$ is maximal.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך