

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

Probability — 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus of

Polynomials, Rational Functions, and

Root Functions

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית,

הסתברות 20×2 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

20×1 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

של פולינומים, של פונקציות

רציונליות ושל פונקציות שורש

20×2 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות

במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון

או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. A group of athletes walks 40 km on a regular route every day.

On Sunday the group set off on a walk. After 3 hours of walking at a constant initial speed, the group stopped for a 15 minute break, and then continued walking until the end of the route at a speed that was 3 km/h faster than the group's initial speed.

On Monday the group walked without stopping. The group walked at a speed that was 60% faster than its initial speed on Sunday.

On both days, the group set off at the same time, but on Monday it arrived at the end of the route one hour earlier than on Sunday.

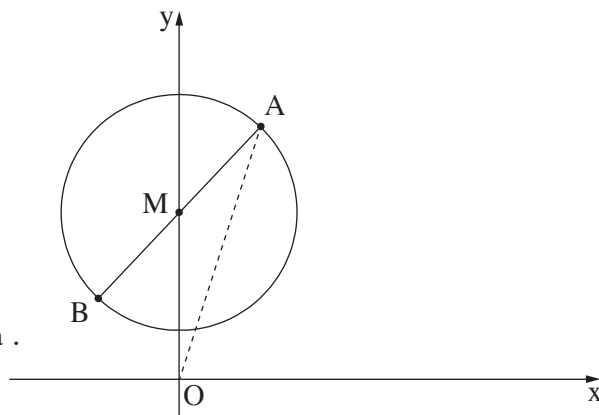
א. Find the group's initial speed on Sunday.

ב. Find how long it took the group of athletes to walk the entire route on Monday.

2. Point M is located on the y-axis, as shown in the figure on the right.

Point A is located on a circle whose center is at Point M.

Given: The equation of Line AO is $y = 3x$
(Point O is the origin),
the equation of Line AM is $y = x + 2a$.
 $a > 0$ is a parameter.



- א. Express the coordinates of Points M and A in terms of a .

Given: The radius of the circle equals $\sqrt{32}$.

- א. Calculate the value of a .

Substitute the value of a that you found, and answer Items ג and ד.

Point B is located on the circle such that AB is a diameter of the circle.

- ג. Find the coordinates of Point B.

A tangent to the circle was drawn at Point A. The tangent intersects the x-axis at Point C.

- ד. (1) Calculate the area of Triangle ABC.
(2) Calculate the area of Quadrilateral ABOC.

3. In a large high school, some of the students have laptops and the other students do not.
If 3 students from the school are chosen at random, the probability that 2 of the three have a laptop is 0.512.

- א. What is the probability that a student (boy or girl) at this school has a laptop?

Given: The number of girls in the school is $1\frac{1}{2}$ times larger than the number of boys.
Half of the students who do not have laptops are boys.

A student at the school (boy or girl) was chosen at random.

- א. What is the probability that the chosen student is a boy who has a laptop?
ב. We know that a girl was chosen. What is the probability that she has a laptop?
ג. 2 students at the school were chosen at random (from among the boys and girls).
What is the probability that at least one of them (boy or girl) has a laptop?

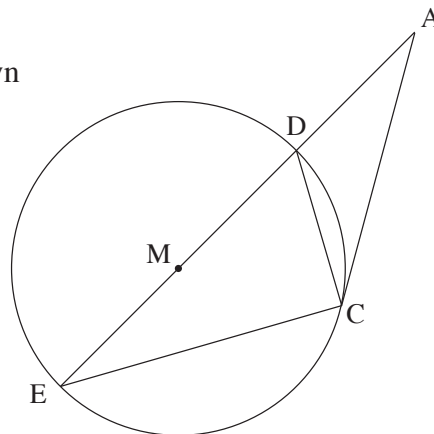
Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: a circle whose center is M.

From Point A, which is outside the circle, a line was drawn tangent to the circle at Point C, and another line was drawn which passes through Point M and intersects the circle at Points D and E, as shown in the figure.



א. Prove: $\triangle ADC \sim \triangle ACE$.

ב. Prove: $\angle ACD = \angle MCE$.

ג. Prove: $\angle MCA = \angle ECD$.

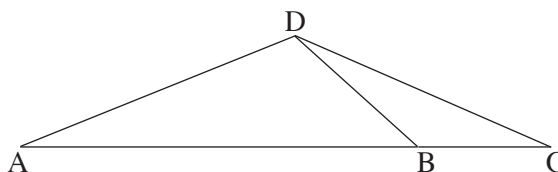
ד. Given: $MD = DA$.

Prove that Triangle MCD is an equilateral triangle.

5. In Triangle ADC, Point B is located on Side AC

(see figure).

Given: $AD = 4$, $\angle ADB = 110^\circ$,
the area of Triangle ADB is 5.



א. Calculate the length of Segment BD.

ב. Find the size of Angle DBA.

Given: The length of the radius of the circle that circumscribes Triangle BDC is 3.

ג. Calculate the length of Side DC.

Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

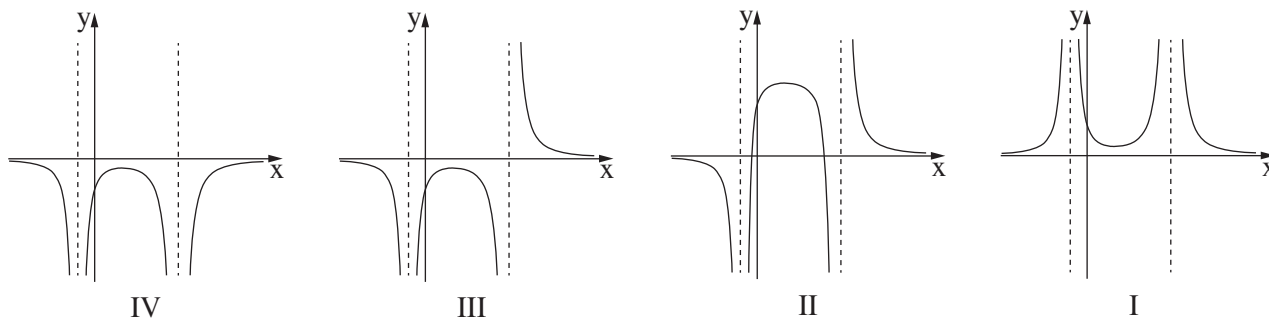
6. Given: the function $f(x) = \frac{x-1}{x^2-4x-a}$. a is a parameter.

Given: One of the vertical asymptotes of function $f(x)$ is $x = -1$.

א. Find a .

Substitute $a = 5$ and answer Items ב-ה.

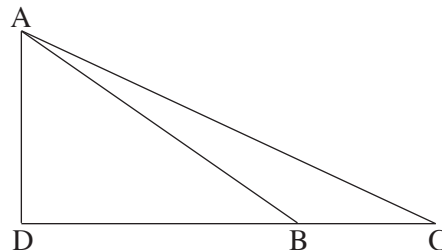
- ב. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
(2) Write the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
(3) Find the intervals on which function $f(x)$ is increasing and decreasing (if there are such intervals).
(4) Find the coordinates of the points at which the graph of function $f(x)$ intersects the axes.
(5) Sketch the graph of the function $f(x)$.
- ג. Find the equations of the asymptotes of the derivative function, $f'(x)$, that are perpendicular to the axes.
- ד. Determine which of the four graphs (I-IV) given at the end of the question is the graph of the derivative function, $f'(x)$. Explain.
- ה. Calculate the area of the region bounded by the derivative function $f'(x)$, the x -axis, and the lines $x = 0$ and $x = 4$.



Continued on page 6 ►

7. Given: the function $f(x) = \sqrt{-2x^4 + 16x^2 + 18}$, whose domain [תחום ההגדרה] is $-3 \leq x \leq 3$.
- א. (1) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what kind of extrema they are. You may leave a root sign in your answer.
 - (2) Find the intervals on which function $f(x)$ is increasing and decreasing.
 - ב. Sketch the graph of the function $f(x)$.
 - ג. How many points of intersection are there between the line $y = 5$ and the graph of function $f(x)$? Explain.
 - ד. Sketch the graph of the function $-f(x)$.

8. Given: a right triangle ADC ($\angle ADC = 90^\circ$).
A line was drawn from Point A to intersect Side DC at Point B, such that $DB = 2BC$.
Given: The area of Triangle ABC is 9.
Let: $BC = x$.



- א. Express the length of the height to Side BC in Triangle ABC in terms of x .
- ב. Find the value of x for which the value of AB^2 is minimal. Explain.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך