The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2020
Exam Number: 035582
Appendix: Formula sheets for 5 points
English Translation (3)

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers
$2\times 33\frac{1}{3} - 66\frac{2}{3}$ points

Part Two: Growth and Decay, Power Functions, Exponential and Logarithmic Functions
$1\times 33\frac{1}{3} - 33\frac{1}{3}$ points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you use any draft paper outside the answer booklet your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תש"ף, 2020
מספר השאלון: 035582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים, טריגונומטריה במרחב, מספרים מרוכבים
$2\times 33\frac{1}{3} - 66\frac{2}{3}$ נקודות

פרק שני: גדילה ודעיכה, פונקציות חזקה, פונקציות מעריכיות ולוגריתמיות
$1\times 33\frac{1}{3} - 33\frac{1}{3}$ נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה מסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Point A is located on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant. a and b are positive parameters.

Given: $a>b$, the length of the major axis of the ellipse is 13.

F_1 and F_2 are the foci of the ellipse.

The perimeter of Triangle F_1AF_2 is 25 and its area is 12.

א. Find the equation of this ellipse.

ב. Find the coordinates of Point A.

A parabola whose equation is $y^2=2px$ ($p>0$ and is not a whole number) passes through Point A.

A tangent to the parabola is drawn through Point A. The tangent intersects the x-axis at Point L.

ג. Find the x-coordinate of Point L.

The parabola and the ellipse intersect at another point, B.

Point D is located on Line AB.

ד. Find the equation of the locus on which all the centroids [נקודות מפגשי התיכונים] of the Triangles ALD are located.

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2. Line ℓ passes through the origin, O, and is perpendicular to Plane π.

Point $P(-1,-1,2)$ is the intersection point of Line ℓ and Plane π.

א. Find the equation of Plane π.

OABCD is a right pyramid whose base, ABCD, is a rectangle located on Plane π (Point O is the origin).

Points A and B are the points of intersection of Plane π with the x-axis and the y-axis, respectively.

ב. **(1)** Find the coordinates of the vertices A and B.

(2) Find the coordinates of the vertices C and D.

ג. Calculate the size of the angle between the side face AOB of the pyramid OABCD and the base of the pyramid.

Points $F(-4,-2,0)$ and $G(-2,-4,0)$ are located on Segment AB.

ד. **(1)** Show that $|FG| = \frac{1}{3}|AB|$.

(2) Find coordinates of two points, H and I, such that the volume of Pyramid OFGHI is $\frac{1}{3}$ of the volume of Pyramid OABCD.

Explain your answer.

3. **א.** Solve the equation $z^3 = -1$ (z is a complex number). Provide detailed calculations.

a_n is a geometric sequence whose common ratio is 2i.

ב. Show that $a_{n+4} = 16a_n$ for any natural n.

Points A, B, C, and D on the Gauss plane represent the terms a_1, a_2, a_3, and a_4 of the sequence, respectively.

a_1 is one of the solutions of the equation $z^3 = -1$.

Point A is located in the first quadrant.

ג. **(1)** Draw a sketch of the quadrilateral ABCD.

(2) Find the area of the quadrilateral ABCD.

ד. Points A', B', C', and D' represent the terms a_5, a_6, a_7, and a_8 of the sequence, respectively.

Find the ratio of the area of the quadrilateral $A'B'C'D'$ to the area of the quadrilateral ABCD $\left(\frac{S_{A'B'C'D'}}{S_{ABCD}}\right)$. Explain.

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Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: the function $f(x)=e^{\left(\frac{a}{x-1}\right)}+c$; a and c are parameters.

א. Find the domain [תחום ההגדרה] of the function f(x).

Given: The horizontal asymptote of the function f(x) is $y=1$;
the function f(x) intersects the y-axis at the point $(0\,,e^{-4})$.

ב. Find the value of c and the value of a .

ג. **(1)** Find the intervals on which the function f(x) is increasing and the intervals on which it is decreasing (if there are such intervals).

(2) Find the positive and the negative intervals of the function f(x) (if there are such intervals).

The function f(x) has a single inflection point at the point where $x=-1$.

ד. **(1)** Sketch a graph of the function f(x).

(2) For which values of k does the line $y=k$ intersect the graph of the function f(x)? Explain.

ה. A tangent to the graph of the function f(x) was drawn at the function's inflection point. The tangent passes through the origin.

Explain why the area located in the second quadrant and bounded by the graph of the function f(x), the tangent, and the y-axis is smaller than $\frac{1}{2}e^{-2}$.

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5. Given: The derivative function of the function f(x) is: $f'(x)=\frac{\ell n(-x)+2}{x}$.

The functions f(x) , $f'(x)$, and $f''(x)$ have the same domain [תחום הגדרה].

א. **(1)** Find the domain of the function f(x).

(2) Find the intervals on which the function f(x) is increasing and on which it is decreasing.

(3) Find the intervals on which the function f(x) is convex [קעירות כלפי מעלה] [$\cup$] and on which it is concave [קעירות כלפי מטה] [$\cap$].

ב. **(1)** What are the equations of the asymptotes of the derivative function $f'(x)$ that are perpendicular to the axes?

(2) Sketch a graph of the derivative function $f'(x)$.

Given: $f(-e^{-2})=0$.

ג. **(1)** Find an algebraic expression for the function f(x).

(2) Sketch a graph of the function f(x).

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך