The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2020
Exam Number: 035581
Appendix: Formula sheets for 5 points
English Translation (3)

Mathematics
5 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra and Probability
– 2 × 20 – 40 points
Part Two: Planar Geometry and Planar Trigonometry
– 1 × 20 – 20 points
Part Three: Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions
– 2 × 20 – 40 points
Total – 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or may lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תש"ף, 2020
מספר השאלון: 035581
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה
5 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה והסתברות
— 2×20 — 40 נקודות
פרק שני: גאומטריה וטריגונומטריה במישור
— 1×20 — 20 נקודות
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות
— 2×20 — 40 נקודות
סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The distance between City A and City B is 96 km.

A car and a truck left City A at the same time and traveled toward City B.

At first, the car traveled at a constant speed of V_1 kph.

After 15 km, it stopped on the side of the road for half an hour, in order to fix a problem.

After the problem was fixed, the car continued on its way at a constant speed of 90 kph.

The truck traveled the entire route at a constant speed of V_2 kph.

It passed the car 3 minutes after the car had stopped on the side of the road.

The car and the truck arrived at City B at the same time.

א. Find V_1 and V_2.

ב. How long after they had set off was the distance between the car and the truck 3 km? (Find two of the three cases.)

/Continued on page 3/

2. a_n is an arithmetic sequence.

k and p are natural numbers. $k < p$.

Given: $a_k = p$, $a_p = k$.

א. **(1)** Prove that the common difference of the sequence a_n is -1.

(2) Express a_1 in terms of k and p.

The sequence c_n is defined as follows: $c_n = a_n - n$.

Given: The sum of the first 6 terms in the sequence c_n is 0.

ב. **(1)** Find a_1 .

(2) What are the values of k and p? Find all the possibilities.

ג. Calculate the sum $(c_1 - c_2)^2 + (c_3 - c_4)^2 + ... + (c_{99} - c_{100})^2$. Explain.

3. There are 12 balls in a box; most of them are blue and the rest are red.

A ball was randomly taken out of the box, returned to the box, and then again a ball was randomly taken out of the box and returned.

The probability that the two balls that were taken out of the box were different colors is $\frac{4}{9}$.

א. Find how many blue balls are in the box.

ב. Yellow balls were added to the box.

A ball was then randomly taken out of the box, returned, and then again a ball was randomly taken out and returned.

The probability that these two balls were different colors remained $\frac{4}{9}$.

How many yellow balls were added to the box?

All the yellow balls were transferred to a different container, and only the blue and red balls were left in the box.

ג. Balls were randomly taken out of the box one after another (without returning them), until a red ball was taken out.

What is the probability that the number of times a ball was taken out of the box was greater than 3?

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Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. AD and CE are angle bisectors in Triangle ABC, and they intersect at Point F.

Given: $\sphericalangle ABC = 60^o$.

א. Prove that it is possible to circumscribe Quadrilateral BDFE in a circle.

Given: FB is a diameter of the circle that circumscribes Quadrilateral BDFE.

ב. Prove that Triangle ABC is an equilateral triangle.

The extension of Segment BF intersects Side AC at Point G.

ג. Prove that Segment FG is equal in length to the radius of the circle that circumscribes Quadrilateral BDFE.

A tangent to the circle that circumscribes Quadrilateral BDFE is drawn through Point F.
The tangent intersects Sides BA and BC at Points K and L, respectively.

ד. Find the ratio $\frac{KL}{AC}$. Explain your answer.

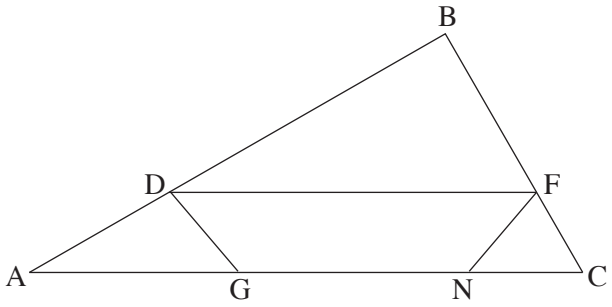
/Continued on page 5/

5. In Triangle ABC, Points D and F are located on Sides BA and BC, respectively, such that $DF \parallel AC$.

Points G and N are located on Side AC such that Quadrilateral DFNG is an isosceles trapezoid, as shown in the figure.

Let: $\sphericalangle BAC = \alpha$, $\sphericalangle FNC = \beta$.

Given: $AD = 7$, $FC = 4$, $\sphericalangle FCN = 2\alpha$.



א. **(1)** Show that $\frac{FN}{\sin\alpha} = \frac{AD}{\sin\beta}$.

(2) Calculate the value of α .

Given: The area of Triangle BDF is 56.

ב. Find the length of Segment DF.

ג. What is the ratio of the radius of the circle that circumscribes Triangle FCN to the radius of the circle that circumscribes Triangle DAG? Explain.

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Part Three—Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

(40 points)

Answer <u>two</u> of the questions 6-8 (each question – 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x)=\dfrac{6}{2\cos^2 x-5\cos x-3}$ on the interval $0\le x\le 2\pi$.

Answer Items א-ג for the given interval.

א. **(1)** Find the domain [תחום ההגדרה] of the function f(x).

(2) Find the coordinates of the extrema [נקודות הקיצון] of the function f(x), and determine what type of extrema they are.

(3) Sketch a graph of the function f(x).

Given: the function $h(x)=\left|f(x)+2\right|$, whose domain is the same as that of function f(x).

ב. **(1)** Sketch a graph of the function h(x).

(2) k is a parameter. Find all the values of k for which the line $y=k$ intersects the graph of the function h(x) at four different points.

Given: the function $g(x)=\left|f(x)\right|+2$, whose domain is the same as that of function f(x).

ג. Is $h(x)<g(x)$ for all values of x in the domain? Explain.

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7. Given: the function $f(x)=\frac{3x}{4x^2-1}$ whose domain [תחום ההגדרה] is $x\neq\pm\frac{1}{2}$.

א. **(1)** Find the intervals on which the function f(x) is increasing and the intervals on which it is decreasing (if there are such intervals).

(2) Find the positive and negative intervals of the function f(x).

Given: the function $g(x)=\sqrt{\frac{3x}{4x^2-1}}$.

ב. **(1)** What is the domain of the function g(x)?

(2) What are the equations of the asymptotes of the function g(x) that are perpendicular to the axes?

Given: The function g(x) has exactly one inflection point. The x-coordinate of this point is less than zero.

ג. **(1)** Sketch a graph of the function g(x).

(2) Sketch a graph of the derivative function, $g'(x)$.

ד. What is the domain of the function $h(x)=\frac{\sqrt{3x}}{\sqrt{4x^2-1}}$?

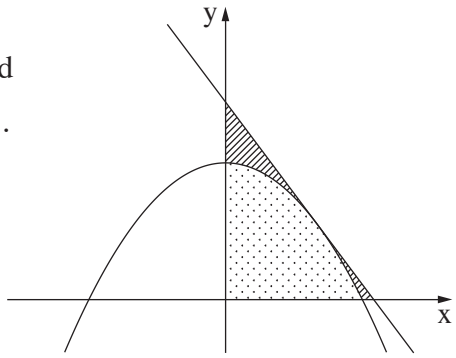
/Continued on page 8/

8. Given: the function $f(x) = -x^2 + 1$.

t is a parameter. Given: $0 < t < 1$.

A tangent to the graph of the function f(x) was drawn at the point where $x = t$ (see figure).

א. Show that the equation of the tangent is $y = -2tx + t^2 + 1$.



Let S denote the dashed area in the figure (the area bounded by the graph of the function f(x), the tangent, and the axes).

ב. Find the value of t for which the area S is minimal.
You may leave a root sign in your answer.

Let A denote the dotted area (the area in the first quadrant bounded by the graph of the function f(x) and the axes).

ג. Determine whether each of the two statements (i-ii), is correct or not. Explain your answer.

i) A value of t exists for which $\frac{A}{S}$ is maximal.

ii) A value of t exists for which $\frac{A}{S}$ is minimal.

Good Luck!

בהצלחה!