The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2020
Exam Number: 035482
Appendix: Formula sheets for 4 points
English Translation (3)

Mathematics
4 Points — 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes.

ב. Exam structure and breakdown of points:
This exam has two parts.

Part One — Sequences, Spatial Trigonometry
— $1 \times 33\frac{1}{3}$ — $33\frac{1}{3}$ points

Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions
— $2 \times 33\frac{1}{3}$ — $66\frac{2}{3}$ points
Total — 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you use any draft paper outside the answer booklet your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תש"ף, 2020
מספר השאלון: 035482
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה
4 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים.
פרק ראשון — סדרות, טריגונומטריה במרחב
— $1 \times 33\frac{1}{3}$ — $33\frac{1}{3}$ נקודות
פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה
— $2 \times 33\frac{1}{3}$ — $66\frac{2}{3}$ נקודות
סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Sequences, Spatial Trigonometry ($33\frac{1}{3}$ points)

Answer one of the questions 1-2.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

Sequences

1. Given: an infinite geometric sequence whose common ratio is $\frac{1}{4}$ and whose sum is $9\frac{1}{3}$.

א. Find the first term of the sequence.

A new term is added between every two adjacent terms in the sequence such that a new infinite geometric sequence is formed whose terms are all positive.

ב. What is the common ratio of the new sequence? Explain.

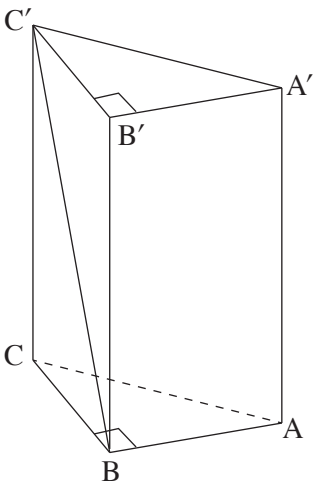
ג. Determine whether each of the following statements, I and II, is true or false.
Explain your answers.

I) The fifth term in the new sequence is equal to the tenth term in the given sequence.

II) The sum of the terms in the even positions in the new sequence is $\frac{1}{2}$ the sum of the given sequence.

Spatial Trigonometry

2. $ABCA'B'C'$ is a right triangular prism (see figure).
The prism base, ABC, is an isosceles right triangle ($\sphericalangle ABC = 90^o$, $AB = BC$).
Given: The height of the prism is 8 and the length of the hypotenuse of the prism base is $4\sqrt{2}$.



א. Find the size of the angle between the diagonal of the side face $BB'C'C$ and the prism base.

ב. Find the size of the angle $AC'B$.

ג. Find the area of the triangle $AC'B$.

Point D is the midpoint of Side CB.

ד. Calculate the length of the segment $A'D$.

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Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions ($66\frac{2}{3}$ points)

Answer <u>two</u> of the questions 3-5 (each question — $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

3. Given: the function $f(x)=\sin 2x$ defined on the interval $0 \le x \le \pi$.

Answer Items א-ג for the interval $0 \le x \le \pi$.

א. **(1)** Find the coordinates of the points of intersection of the graph of the function f(x) with the axes.

(2) Find the coordinates of the extrema [נקודות הקיצון] of the function f(x), and determine what type of extrema they are.

(3) Sketch a graph of the function f(x).

Given: the function $g(x)=2\sin x$. Like f(x), this function is defined on the interval $0 \le x \le \pi$.

ב. Find the coordinates of the points of intersection of the graph of the function g(x) with the graph of the function f(x).

Given: Except for the points you found in Item ב, the graph of function g(x), on the given interval, is located above the graph of function f(x).

ג. Calculate the area bounded by the graphs of functions f(x) and g(x) on the given interval.

4. Given: the function $f(x)=9-(\ell nx)^2$.

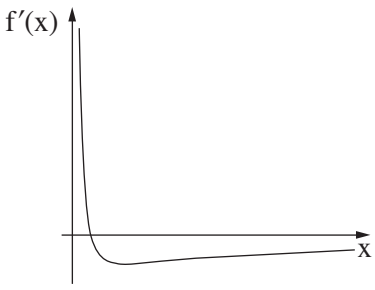
א. **(1)** What is the domain [תחום ההגדרה] of the function f(x)?

(2) Find the coordinates of the intersection points of the graph of the function f(x) with the axes (if there are any).

(3) Find the coordinates of the extremum [נקודת הקיצון] of the function f(x), and determine what type of extremum it is.

(4) Sketch a graph of the function f(x).

The figure on the right describes the graph of the derivative function $f'(x)$.



ב. Calculate the area bounded by the graph of the derivative function $f'(x)$, the line $x=e$, and the x-axis.

/Continued on page 4 /

5. The quantity of a radioactive substance decays exponentially.

An initial amount of this substance was measured on a certain day.

One year after the first measurement, $\left(\frac{1}{3}\right)^2$ kg of the substance remained.

Five years after the first measurement, $\left(\frac{1}{3}\right)^{10}$ kg of the substance remained.

א. Find the substance's rate of decay and the initial amount of substance.

Given: the function $g(x)=3^{-2x}$ which is defined for any $x \geq 0$.

The function g(x) describes the amount of substance x years after the first measurement.

ב. **(1)** Find the coordinates of the points of intersection of the graph of the function g(x) with the axes (if there are any).

(2) Find the intervals on which the function g(x) is increasing and the intervals on which it is decreasing (if there are any).

(3) Sketch a graph of the function g(x) for $x \geq 0$.

Good Luck!