

**The State of Israel
Ministry of Education**

Exam Type: *Bagrut*

Exam Date: Winter 2020

Exam Number: 035481

Appendix: Formula sheets for 4 points

English Translation (3)

מדינת ישראל

משרד החינוך

סוג הבחינה: בגרות

מועד הבחינה: חורף תש"ף, 2020

מספר השאלון: 035481

נספח: דפי נוסחאות ל-4 יחידות לימוד

תרגום לאנגלית (3)

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

Probability — 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus of

Polynomials, Rational Functions, and

Root Functions

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות

2×20 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

1×20 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים,

של פונקציות רציונליות ושל פונקציות שורש

2×20 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The price of an adult entrance ticket to a museum is x shekels.

An adult ticket is twice the price of a child ticket.

The price of a student ticket is 25% lower than an adult ticket.

⌘. Express the price of a child ticket and the price of a student ticket in terms of x .

On Sunday, only adults visited the museum.

The museum's total revenue from ticket sales on Sunday was 1,560 shekels.

On Monday, only children and students visited the museum. The number of children who visited the museum on Monday was 16 more than the number of adults who had visited on Sunday.

The number of students who visited the museum on Monday was smaller by 2 than the number of children who visited it the same day.

The museum's total revenue from ticket sales on Monday was 2,912 shekels.

- ⌘. (1) Find the price of an adult entrance ticket to the museum.
(2) By what percentage was the number of visitors on Monday greater than the number of visitors on Sunday?

2. In Triangle ABC, the triangle vertices B and C are located on the y-axis, as shown in the figure. The equation of the line CA is $y = 5$, and the equation of the line BA is $y = 2x - 3$.

⌘. Find the coordinates of Points C, B, and A.

Given: Point E is in the second quadrant, and its y-coordinate is 1.

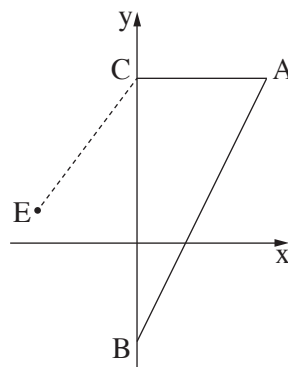
The length of Segment CE is 5.

- ⌘. Find the x-coordinate of Point E.

Point D is the center of the circle that circumscribes Triangle ABC.

- ⌘. Find the equation of the circle that circumscribes Triangle ABC.

- ⌘. Is Point E located on, inside or outside the circle that circumscribes Triangle ABC?
Explain and show detailed calculations.



3. 80% of the 11th-grade students at a large school went on a trip.

There are both boys and girls in the 11th grade.

We know that 0.75 of the boys and $\frac{5}{6}$ of the girls in that grade went on the trip.

An 11th-grade student was selected at random.

- א. (1) What is the probability that a girl was selected?
(2) What is the probability that a girl who went on the trip was selected?
- ב. We know that the student who was selected had been on the trip. What is the probability that a girl was selected?
- ג. 5 students were selected at random from among the boys and girls of the 11th grade. What is the probability that exactly 3 of them were boys who had been on the trip?

Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

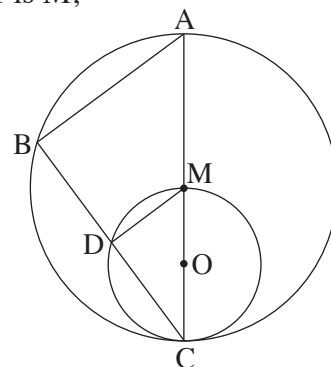
Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. The figure to the right shows two circles: a large circle whose center is M, and a small circle whose center is O.

The small circle is internally tangent to the large circle at Point C and passes through Point M (see figure).

Segment CM passes through Point O, and its extension intersects the large circle at Point A.

An additional line, intersecting the circles at Points B and D, was drawn through Point C, as shown in the figure.



א. (1) Prove that $\angle ABC = \angle MDC$.

(2) Prove that $\triangle ABC \sim \triangle MDC$.

ב. (1) Prove that DM is a midsegment of Triangle ABC.

(2) What is the ratio between the area of Triangle ABC and the area of Triangle MDC? Explain.

ג. Given: $CO = 2$, $DM = 2.4$.

Calculate the length of Segment BC.

5. In Triangle ABD, Point C is located on Side BD (see figure).

Given: $AC = 7$, $CD = 4$, $AD = 10$.

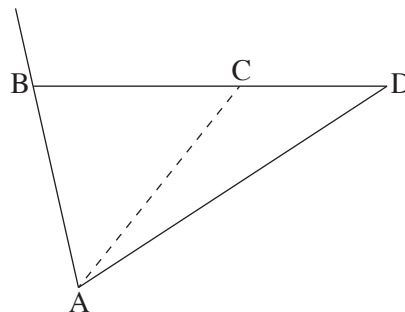
א. Calculate the size of Angle ACD.

Given: $AB = BC$.

ב. Calculate the area of Triangle ABD.

Point E is located on the extension of Side AB, such that the area of Triangle EBD is 4 times smaller than the area of Triangle ABD.

ג. What is the length of Side EB? Explain.



Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = \frac{3}{x^2} - 6x$.

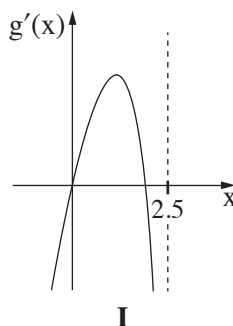
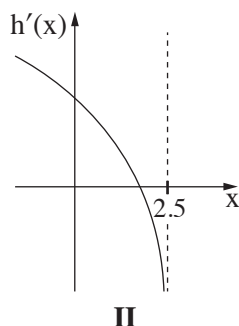
- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$ and determine what type of extremum it is.
- (3) Find the coordinates of the intersection point of the graph of the function $f(x)$ with the x -axis.
Your answer should be correct to two decimal places.
- (4) Sketch a graph of the function $f(x)$.

Given: the function $g(x) = f(x) + c$. c is a parameter.

Given: The extremum of the function $g(x)$ is located on the x -axis.

- ב. (1) Find c .
- (2) Sketch a graph of the function $g(x)$.
- ג. Calculate the area of the region that is bounded by the graph of function $g(x)$, the line $x = -3$, and the x -axis.

7. The graphs shown below are the graphs of the functions $g'(x)$ and $h'(x)$, which are the derivative functions of the functions $g(x)$ and $h(x)$, respectively. The derivative functions $g'(x)$ and $h'(x)$ are defined on the domain $x < 2.5$.



- א. Use the graphs to determine how many internal extrema [נקודות קיצון] the functions $g(x)$ and $h(x)$ have on the domain $x < 2.5$. Explain your answer. (In your answer discuss the part of the graph shown in the figure.)

The function $f(x) = 3 + x^2 \cdot \sqrt{5 - 2x}$ is defined on the domain $x \leq 2.5$.

- ב. Find the coordinates of all the extrema of the function $f(x)$, and determine what type of extrema they are.
- ג. Sketch a graph of the function $f(x)$.
- ד. One of the graphs shown in the beginning of the question (I-II) is the graph of the derivative function $f'(x)$. Determine which one is the graph of $f'(x)$. Explain.
- ה. Calculate the area bounded by the graph of the derivative function $f'(x)$ and by the x -axis in the first quadrant.

8. The graph of the function $f(x) = -x^2 + 4x$ passes through the origin O, and intersects the x-axis at another point, C (see figure).

א. Find the coordinates of Point C.

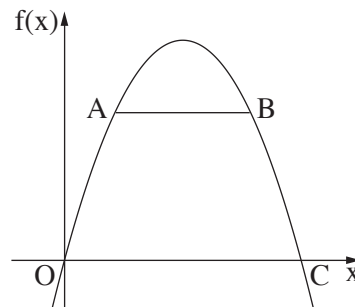
Points A and B are located on the graph of the function $f(x)$ in the first quadrant, as shown in the figure.

Let x denote the x-coordinate of Point A.

Given: the x-coordinate of Point B equals $(4 - x)$.

ב. Explain why Line AB is parallel to the x-axis.

ג. Find the x-coordinate of Point A for which the area of Trapezoid OABC is maximal.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך