

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors,

Spatial Trigonometry, Complex Numbers

$$2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ points}$$

Part Two: Growth and Decay, Power Functions,

Exponential and Logarithmic Functions

$$1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ points}$$

$$\text{Total} \quad \text{---} \quad 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.

Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or

lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

$$66 \frac{2}{3} - 33 \frac{1}{3} \times 2 \text{ נקודות}$$

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

$$33 \frac{1}{3} - 33 \frac{1}{3} \times 1 \text{ נקודות}$$

$$\text{סה"כ} \quad \text{---} \quad 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה מסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טייטה" בראש כל עמוד המשמש טייטה. כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Given: a circle whose equation is $x^2 + y^2 = a^2$. a is a positive parameter.

The circle was shifted horizontally [הוזזה אופקית] to the right so as to be tangent to the y-axis.

א. Express in terms of a the equation of the circle that was obtained.

- ב. Another circle is constructed that is externally tangent to the circle obtained in Item א and is also tangent to the y-axis. The x-coordinate of the center of the constructed circle is positive.

Find the equation of the locus [המקום הגאומטרי] on which the centers of the circles that are constructed in this way are located (if necessary, use a).

The line $y = x + 3$ is tangent at Point M to the locus whose equation you found in Item ב.

- ג. Find the value of a .

- ד. Find the coordinates of the point of tangency of the following two circles:

I. The circle obtained in Item א.

II. The circle constructed as described in Item ב and whose center is at Point M.

2. Given: Pyramid SABCD, whose base, ABCD, is square (see figure).

Point O is the point of intersection of the base diagonals.

Point P is a point on Segment SD, and it satisfies $\vec{SP} = t \cdot \vec{SD}$, $t > 0$.

Let us denote: $\vec{AB} = \underline{u}$, $\vec{AD} = \underline{v}$, $\vec{AS} = \underline{w}$.

א. Express the vector $\vec{OP}$ in terms of $\underline{u}$, $\underline{v}$, $\underline{w}$ and t .

ב. Find the value of t for which OP is parallel to the plane of Face SAB.

Given: The side length of Square ABCD is 4,

AS is perpendicular to the base of the pyramid, and

$$AS = 4\sqrt{2}.$$

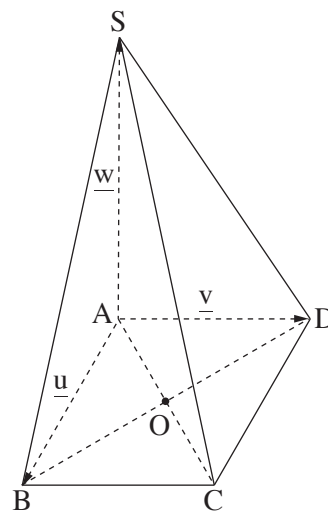
Point A is the origin.

Points B, D, and S are located on the positive part of the x, y, and z axes, respectively.

ג. Find the values of t for which Line OP forms a 45° angle with the plane of Face SAD.

Point T is located on Segment SC, such that TABCD is a right pyramid.

ד. Find the volume of Pyramid TABCD.



3. Given: a geometric sequence whose first term is 1 and whose second term is iz (z is a complex number).

Given: The sequence is not constant.

א. (1) Write down the first five terms of the sequence (if necessary, express in terms of z).

(2) Prove that the sum of the first five terms of the sequence equals $\frac{z^5 + i}{z + i}$.

ב. (1) Find all the solutions of the equation $z^5 = -i$ (z is a complex number).

(2) Find all the solutions of the equation $1 + iz - z^2 - iz^3 + z^4 = 0$ (z is a complex number).

Point A is located in the third quadrant of the Gaussian plane, and it corresponds to one of the solutions of the equation you solved in Sub-item ב(2).

ABO is an equilateral triangle in the Gaussian plane (O is the origin).

ג. Find the complex number corresponding to Point B (find both possibilities).

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: the function $f(x) = \ln(x^2 + ax + 1)$.

a is a parameter, $-2 < a < 2$.

- א. Show that the function $f(x)$ is defined for all x .
- ב. Find the coordinates of the points of intersection of the graph of the function $f(x)$ with the x -axis (if necessary, express in terms of a).
- ג. Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$ and determine what type of extremum it is (if necessary, express in terms of a).
- ד. Three graphs (I-III) are shown at the end of the question that describe the graph of the function $f(x)$ depending on the parameter a .

Each of the graphs corresponds to one of the ranges (1)-(3) of a :

(1) $0 < a < 2$

(2) $-2 < a < 0$

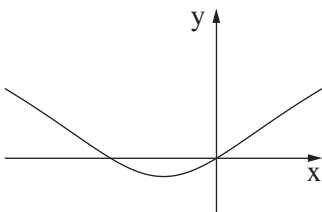
(3) $a = 0$

Write down which of the value ranges (1)-(3) corresponds to each of the graphs.

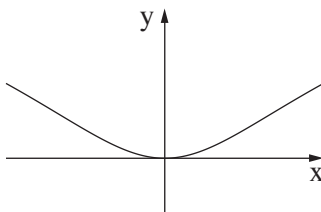
Answer Item ה for a that satisfies $-2 < a < 0$.

Let S denote the area of the region bounded by the graph of the function $f(x)$ and the x -axis.

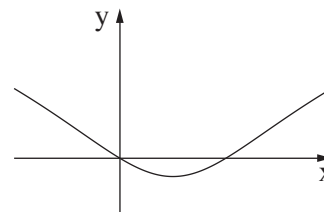
ה. Express the integral $\int_0^{-a} \ln(4x^2 + 4ax + 4) dx$ in terms of a and S .



I

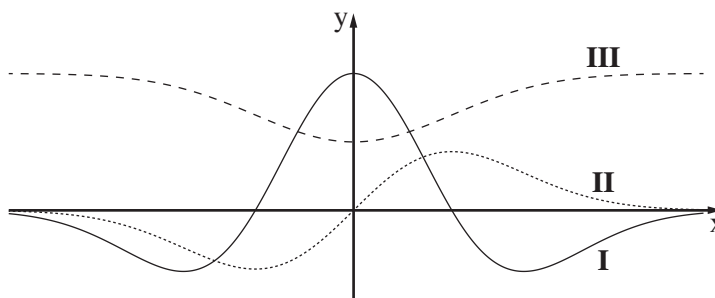


II



III

5. The figure below shows the graphs of the functions f , f' , f'' , which are defined for all x .
 Graph III is located entirely above Graph II.



- א. Match each one of the graphs I, II, III with function f , f' , or f'' .

Explain your answer.

Point A is located on the graph of the function $f(x)$, and Point B is located on the graph of the function $f'(x)$, such that Segment AB is parallel to the y-axis.

Given: $f'(x) = x \cdot e^{-x^2}$.

- ב. Find the value of x for which the length of Segment AB is minimal, and
 find the value of x for which the length of Segment AB is maximal.

Given: The maximal length of Segment AB equals $1 + \frac{1}{2e}$.

- ג. Find the function $f(x)$.

Good Luck!

All rights reserved by the State of Israel.
 Copying or publication without the consent of the
 Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך