The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2019, ***Moed Bet***
Exam Number: 035581
Appendix: Formula sheets for 5 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשע"ט, 2019, **מועד ב**
מספר השאלון: 035581
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

Mathematics
5 Points — 1st Exam

מתמטיקה
5 יחידות לימוד - שאלון ראשון

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra and Probability
– 2 × 20 – 40 points
Part Two: Planar Geometry and Planar Trigonometry
– 1 × 20 – 20 points
Part Three: Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions
– 2 × 20 – 40 points
Total – 100 points

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה והסתברות
— 2×20 — 40 נקודות
פרק שני: גאומטריה וטריגונומטריה במישור
— 1×20 — 20 נקודות
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות
— 2×20 — 40 נקודות
סה"כ — 100 נקודות

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or may lead to the disqualification of your exam.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

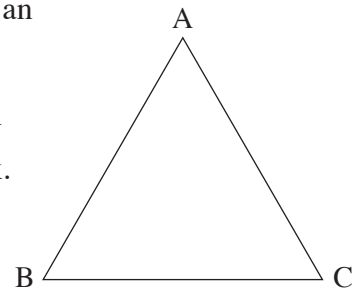
Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The figure at the right shows a bicycle riding track in the shape of an equilateral triangle ABC, with a side length of a meters.



One day, two cyclists set off at the same time from Point A toward Point B. They rode in the same direction along the triangular track. Each of them rode at a constant speed. Cyclist **א**'s speed was 2 meters per second faster than Cyclist **ב**'s speed.

At the same time that Cyclist **א** reached Point A after having cycled around the entire track twice, Cyclist **ב** reached Point B for the second time.

א. Find the speed of each of the cyclists.

ב. What point on the triangle will Cyclist **ב** have reached, when Cyclist **א** reaches Point A after completing the triangular track 5 times?

When Cyclist **א** reached Point A after completing the track 5 times, he turned around and began to ride in the opposite direction, from Point A to Point C, without changing his speed. Cyclist **ב** continued to ride in his original direction, without changing his speed. The cyclists met at Point M.

ג. On what side of the triangle is Point M located? Find the ratio between the two parts into which Point M divides that side.

The next day, the two cyclists again set off from Point A. They rode toward Point B, and then continued to ride along the triangular track, each riding at the same speed as on the previous day. Cyclist **א** passed Cyclist **ב** for the first time 6 minutes after they set off.

ד. Find the perimeter of the triangle. Explain your answer.

/Continued on page 3/

2. Given: a sequence a_n that satisfies the rule $a_{n+1} + a_n = 6n + 5$ for all n.

א. Prove that $a_{n+2} = a_n + c$ is satisfied (c is a constant number), and find the value of c.

ב. Write an example of a sequence a_n that satisfies the rule and is not an arithmetic sequence (write at least the first 4 terms in the sequence).

Given: The entire sequence a_n is arithmetic.

ג. Calculate the value of a_1.

A new sequence of $2n + 1$ terms was constructed:

$a_1 - 1$, $a_2 - 2$, $a_3 - 3$, ... , $a_{2n+1} - (2n + 1)$

The middle term in the new sequence is 43.

ד. Calculate the sum of the terms of the new sequence.

3. A box contains 12 blue balls, 20 red balls, and 8 yellow balls.

28 balls are marked **1** and the rest are marked **0**.

$\frac{1}{4}$ of the balls marked **1** are yellow.

The number of red balls marked **1** is 4 times greater than the number of blue balls marked **0**.

Danny took a ball at random from the box.

א. What is the probability that the ball Danny took was a blue ball marked **1**?

ב. If we know that Danny took at random a blue ball or a ball marked **1**, what is the probability that he took a ball marked **0**?

Danny returned the ball to the box, and now he is playing a game: he takes a ball at random from the box, records the number marked on the ball, and returns the ball to the box.
Each time that he takes a ball marked **1**, he gets a point. He stops playing when he has 5 points.

ג. What is the probability that Danny will have 5 points after exactly 6 rounds?

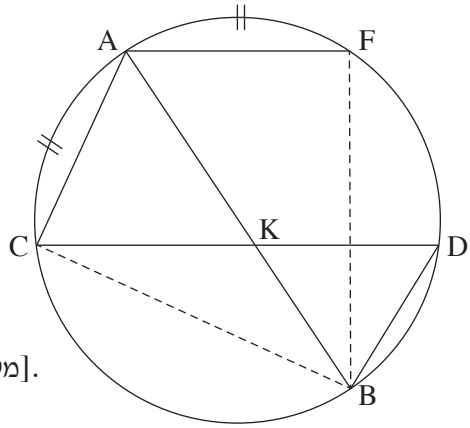
/Continued on page 4/

Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. AB is a diameter of a circle. CD and AF are two chords [מיתרים] of the circle that are parallel to each other.
AB and CD intersect at Point K (see figure).
Given: $\overset{\frown}{CA} = \overset{\frown}{AF}$ (the arcs marked in the figure).



א. **(1)** Prove that $\sphericalangle FAB = \sphericalangle CAB$.

(2) Prove that $BK = BD$.

ב. Prove that the quadrilateral AFKC is a rhombus [מעוין].

ג. Given also: $BD \cdot AB = CD \cdot AC$.

(1) Prove that $\triangle BDC \sim \triangle CAB$.

(2) Prove that CD is a diameter of the circle.

5. Given: Rectangle ABCD. Point E is located on Diagonal AC (see figure).
Given: $\sphericalangle DAC = \alpha$,
$\sphericalangle ADE = \beta$.
R_1 is the radius of the circle that circumscribes Rectangle ABCD.
R_2 is the radius of the circle that circumscribes Triangle ADE.

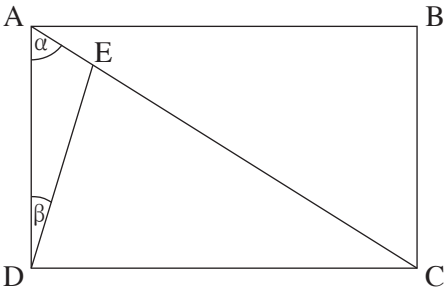
א. Express the ratio $\frac{R_1}{R_2}$ in terms of α and β .

ב. Show that when $\alpha = \beta$, $\frac{R_1}{R_2} < 2$.

ג. Given: $\alpha = 60^o$, $\beta = 15^o$.

(1) Show that $\triangle DEC$ is an isosceles triangle.

(2) Express BE^2 in terms of R_1 .



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Part Three—Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

(40 points)

Answer two of the questions 6-8 (each question – 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = a \cdot \cos 2x + \sin^2 x$, which is defined on the domain $-\pi \le x \le \pi$.
a is a parameter.

א. Is the function f(x) even or odd or neither even nor odd? Explain.

ב. What are the coordinates of the extrema [נקודות הקיצון] of the function f(x) (express in terms of a, if necessary), if it is given that the function is not constant? Determine the types of the extrema according to the value of a (for both possibilities for a).

ג. Find the value of a for which the function f(x) is constant. Explain.

Given: $a > 1$.

ד. **(1)** Sketch the graph of the function f(x).

(2) Sketch the graph of the derivative function $f'(x)$.

ה. Given: The area bounded by the graph of the derivative function $f'(x)$ and the x-axis equals 12. Find the value of a.

7. Given: a circle with Diameter AB. The radius of the circle is 10.
Point P is located on Diameter AB, between the center of the circle and Point B.
A perpendicular to AB is drawn through Point P, intersecting the circle at Points C and D.
Find the maximum area of Triangle ACD.

/Continued on page 6/

8. Given: the function $f(x)=\frac{x^2+bx-c}{x^2-4}$. b and c are parameters.

א. Find the domain [תחום ההגדרה] of the function f (x).

Given: The function f(x) is even.

ב. Find the value of b .

Given: The graph of the function f(x) intersects the x-axis at two points between its two vertical asymptotes.

ג. Find the range of values of c.

ד. **(1)** Find the coordinates of the extremum [נקודת הקיצון] of the function f(x), and determine what type of extremum it is (express in terms of c , if necessary).

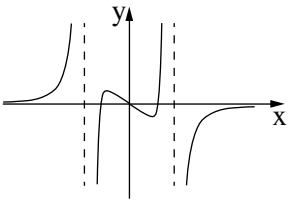
(2) Find the horizontal asymptote of the function f(x), and sketch the graph of the function f(x).

ה. Given: the function $g(x)=f(x)\cdot f'(x)$, which is defined on the same domain as the functions f(x) and $f'(x)$.

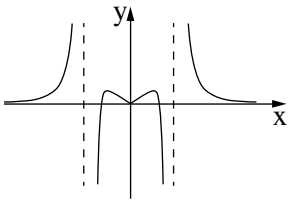
Graphs I-III are shown below.

(1) Which of the graphs, I-III, is the graph of the function g(x)? Explain.

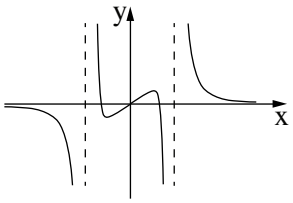
(2) Express in terms of c the area of the region bounded by the graph of the function g(x) and the x-axis.



III



II



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Good Luck!

בהצלחה!