

Mathematics

4 Points — 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes

ב. Exam structure and breakdown of points:

This exam has two parts.

Part One — Sequences, Spatial Trigonometry

$$- 1 \times 33\frac{1}{3} - 33\frac{1}{3} \text{ points}$$

Part Two — Growth and Decay, Differential and Integral

Calculus of Trigonometric Functions,

Exponential and Logarithmic Functions, and

Power Functions

$$- 2 \times 33\frac{1}{3} - 66\frac{2}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

4 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון — סדרות, טריגונומטריה במרחב

$$- 33\frac{1}{3} \times 1 - 33\frac{1}{3} \text{ נקודות}$$

פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של

פונקציות טריגונומטריות,

פונקציות מעריכיות ולוגריתמיות

ופונקציות חזקה

$$- 33\frac{1}{3} \times 2 - 66\frac{2}{3} \text{ נקודות}$$

$$\text{סה"כ} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיטה" בראש כל עמוד המשמש טיטה. כתיבת טיטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Sequences, Spatial Trigonometry ($33\frac{1}{3}$ points)

Answer one of the questions 1-2.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

Sequences

1. Given: two arithmetic sequences, a_n and b_n .

Given: $a_1 = b_1$.

The common difference of sequence a_n is d ,

and the common difference of sequence b_n is $d + 1$.

Given: a_4 is larger than b_3 by 2.

א. Find the value of d .

ב. Show that: $b_n = a_n + n - 1$.

There are n terms in each of the sequences a_n and b_n .

ג. Express, in terms of n , the difference between the sum of all the terms in sequence b_n and the sum of all the terms in sequence a_n .

Given: The difference between the sum of all the terms in sequence b_n and the sum of all the terms in sequence a_n is 780.

The sum of all the terms in sequence a_n is 3,040.

ד. Find the value of a_1 .

Spatial Trigonometry

2. Given: Right pyramid $SABCD$, whose base is square.

Given: The area of the pyramid's base is $4a^2$,

a is a positive parameter.

א. Express the length of the diagonal of the base in terms of a .

Given: The angle between a side edge and the pyramid's base is 67° .

ב. Express the height of the pyramid, SO , in terms of a .

Given: The volume of the pyramid is 15.

ג. (1) Calculate the value of a .

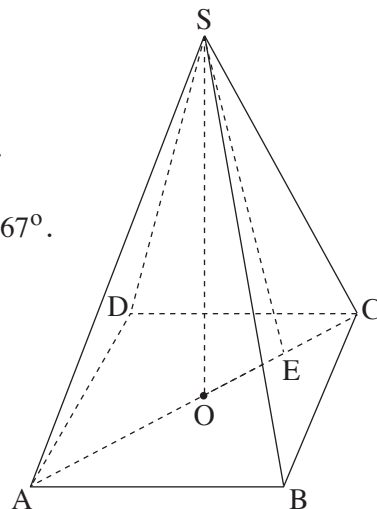
(2) Point E is the midpoint of Segment OC .

Calculate the area of Triangle ASE .

Give your answers correct to 2 decimal places.

ד. The midpoints of the base sides were connected to the pyramid's apex, S , forming a new right pyramid.

Calculate the volume of the new pyramid.



Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions (66 $\frac{2}{3}$ points)

Answer two of the questions 3-5 (each question — 33 $\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

3. Given: the function $f(x) = 2 - \cos^2 x$, on the domain $-\frac{2\pi}{3} \leq x \leq \frac{2\pi}{3}$.

- א. Find the coordinates of all the extrema [נקודות הקיצון] of the function $f(x)$ on the given domain, and determine what kind of extrema they are.
- ב. Does the graph of the function $f(x)$ intersect the x -axis? Explain.
- ג. Is the function even or odd or neither even nor odd? Explain.
- ד. Sketch the graph of the function $f(x)$.
- ה. Given: the function $g(x) = f(x) + c$, c is a parameter.
For which values of c is the graph of the function $g(x)$ tangent to the x -axis?
Find the two possible values.

4. Given: the functions $f(x) = e^{2x-1} - 1$, $g(x) = e^{2-x} - 1$.

$f(x)$ and $g(x)$ are defined for all x .

- א. (1) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes.
(2) Prove that the function $f(x)$ increases for all x .
(3) Find the horizontal asymptote of the function $f(x)$.
- ב. (1) Find the coordinates of the points at which the graph of the function $g(x)$ intersects the axes.
(2) Prove that the function $g(x)$ decreases for all x .
(3) Find the horizontal asymptote of the function $g(x)$.
- ג. On a single set of axes, sketch the graphs of the functions $f(x)$ and $g(x)$.
- ד. Find the coordinates of the point of intersection of the functions $f(x)$ and $g(x)$.
- ה. Calculate the area of the region bounded by the graphs of the functions $f(x)$ and $g(x)$ and the x -axis. You may leave e in your answer or give an answer that is correct to 3 decimal places.

5. Given: the function $f(x) = \frac{1 + \ln x}{ax}$, $a > 0$ is a parameter.
- Find the domain [תחום ההגדרה] of the function $f(x)$.
 - Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes (if there are any such points).
 - Find the x-coordinate of the extremum [נקודת הקיצון] of the function $f(x)$ and determine what kind of extremum it is.
 - Find the intervals on which the function $f(x)$ is increasing and decreasing.

Given: The y-coordinate of the extremum of the function $f(x)$ is $\frac{1}{4}$.

- (1) Find the value of a .
- (2) Sketch the graph of the function $f(x)$.

Given: the function $g(x) = -f(x)$.

- Write down the coordinates of the extremum of the function $g(x)$ and determine what kind of extremum it is. Explain your answer.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך