

**The State of Israel
Ministry of Education**

Exam Type: *Bagrut*

Exam Date: Summer 2019, *Moed Bet*

Exam Number: 035481

Appendix: Formula sheets for 4 points

English Translation (3)

מדינת ישראל

משרד החינוך

סוג הבחינה: בגרות

מועד הבחינה: קיץ תשע"ט, 2019, מועד ב

מספר השאלון: 035481

נספח: דפי נוסחאות ל-4 יחידות לימוד

תרגום לאנגלית (3)

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry, Probability

— 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus of

Polynomials, of Rational Functions, and of
Root Functions

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet.

Write "טיטה" at the top of each draft page.

If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית,

הסתברות — 20×2 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

— 20×1 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

של פולינומים, של פונקציות רציונליות
ושל פונקציות שורש

— 20×2 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד.

רשום "טיטה" בראש כל עמוד המשמש טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Two cyclists set off at 8:00 from Point A.

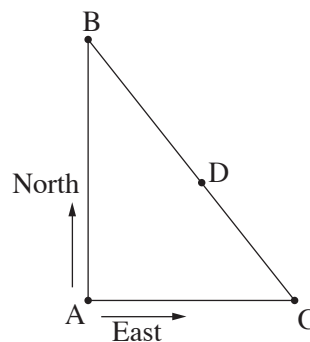
Cyclist א rode northward, and Cyclist ב rode eastward (see figure).

At 9:00, Cyclist א reached Point B and Cyclist ב reached Point C, and the distance between them, BC, was 30 km.

Cyclist א's speed was 6 kilometers per hour faster than

Cyclist ב's speed.

- א. Find the speed of each of the two cyclists.



After a 10-minute break, the cyclists set off toward each other:

Cyclist א rode toward Point C at the same speed as before, and Cyclist ב rode toward Point B at a speed that was 3 kilometers per hour faster than before.

They met at Point D (see figure).

- ב. At what time did the cyclists meet each other?

2. Given: Triangle ABC.

Vertices B and C are located on the x-axis, as shown in the figure below.

Vertex A is located in the first quadrant.

The equation of Side AC is: $y = -4\frac{1}{2}x + 36$.

Given: The length of Side BC is 5.

א. Find the coordinates of Points B and C.

Given: The area of Triangle ABC is $22\frac{1}{2}$.

ב. Find the coordinates of Point A.

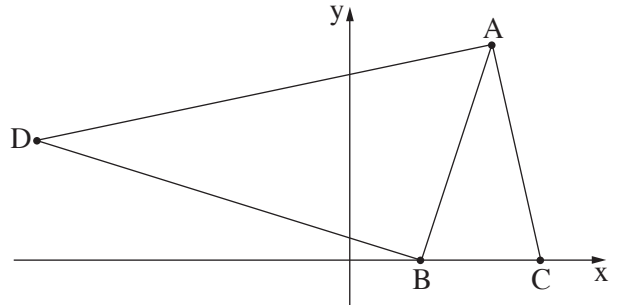
D is a point in the second quadrant, such that DB is perpendicular to AB.

ג. Find the equation of the line BD.

Given: The x-coordinate of Point D is -12.

ד. (1) Prove that $\angle DAC = 90^\circ$.

(2) Find the center of the circle that circumscribes Triangle DAC.



3. A bag contains 80 balls, some made of glass, and the rest made of plastic.

20 of the balls in the bag are blue and the rest are yellow.

70% of the balls in the bag are yellow plastic balls.

25% of the glass balls are yellow.

א. How many plastic balls are in the bag?

ב. A ball was removed from the bag at random and then returned to the bag.

(1) What is the probability that this ball was a blue glass ball?

(2) We know that the ball was blue. What is the probability that it was made of glass?

ג. A ball was removed from the bag at random and then returned to the bag.

This procedure (removing and returning a ball) was performed 4 times.

What is the probability that exactly 3 of the balls removed in this way were yellow?

Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

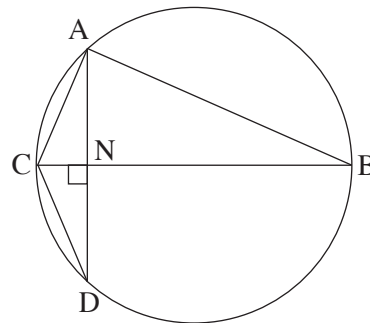
Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Triangle ABC is inscribed in a circle, such that BC is a diameter of the circle.

A perpendicular was drawn from Vertex A to Side BC.

The perpendicular intersects Side BC at Point N and

intersects the circle at Point D, as shown in the figure at right.



א. Prove: $\triangle ABC \sim \triangle NDC$.

ב. Prove: $\triangle ACD$ is an isosceles triangle.

ג. Prove: $AC^2 = NC \cdot BC$.

ד. Given: $CD = 4$, and the radius of the circle is 5. Calculate the length of Segment NC.

5. In the given trapezoid ABCD ($AB \parallel DC$) shown in the figure:

$BD = 6$, $DC = 7$, $BC = 4$.

א. Calculate the size of the angle $\angle BDC$.

Given: $AB = AD$.

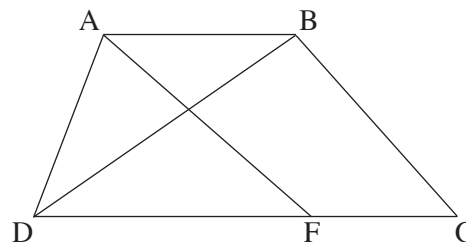
ב. Calculate the length of Side AD.

Point F is located on Side DC.

Given: The area of Triangle ADF is 8.

ג. (1) Find the length of Side DF.

(2) Find the length of the radius of the circle that circumscribes Triangle ADF.



Part Three — Differential and Integral Calculus of Polynomials, of Rational Functions, and of Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = \frac{x^2}{x^2 + 2x - 3}$.

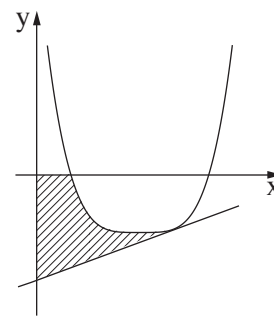
- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the asymptotes of function $f(x)$ that are perpendicular to the axes.
- ב. Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
- ג. Sketch the graph of function $f(x)$.
- ד. (1) Sketch the graph of the derivative $f'(x)$ on the interval $-3 < x < 1$.
- (2) Based on the sketch from Sub-item ד(1), calculate the area of the region bounded by the graph of the derivative $f'(x)$, the x -axis, and the line $x = -2$.

7. Shown at the right is a diagram of the graph of the function $f(x) = (x - 3)^4 - 16$, which is defined for all x .

- א. Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$.
- ב. Find the coordinates of the points at which the graph of function $f(x)$ intersects the x -axis.

A tangent was drawn to the graph of the function $f(x)$ at the point where $x = 4$.

- ג. (1) Find the equation of the tangent.
- (2) Calculate the area of the region bounded by the graph of the function, the tangent, the x -axis, and the y -axis (the shaded area in the figure).



8. In Rectangle ABCD, the sum of the lengths of two adjacent sides is 20.

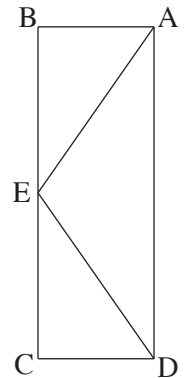
A triangle AED is constructed inside the rectangle, such that Vertex E is located at the midpoint of Side BC (see figure).

Let x denote the length of Segment BE.

- א. (1) Express the length of Segment AE in terms of x .
- (2) Find the lengths of the sides of the rectangle for which the length of Segment AE is minimal (smallest).

Answer Item ב for the rectangle side lengths you found in Item א.

- ב. Calculate the area of Triangle AED.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך