

Mathematics

3 Points — 3rd Exam

Instructions for examinees

א. Duration of the exam: Two hours

ב. Exam structure and breakdown of points:

This exam has six questions in the topic areas:

Algebra, Differential and Integral Calculus.

You must answer four questions — $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

- (2) Formula sheets (attached)

- (3) A Hebrew-foreign language/
foreign language-Hebrew dictionary

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write in the answer booklet only. Write "טייטה" at the top of every page you use for rough work. If you write draft material outside the answer booklet, your exam may be disqualified.

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שעותיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות — $4 \times 25 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

- (2) דפי נוסחאות (מצורפים).

- (3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד.

רשום "טייטה" בראש כל עמוד המשמש טייטה.

כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-6 (each question – 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. A store-owner bought two kinds of hats: straw hats and cloth hats.

He paid 20 shekels for each straw hat, and for each cloth hat he paid 70% more than he paid for a straw hat.

The store-owner bought 120 hats, for which he paid a total of 2,946 shekels.

- א. (1) How much did the store-owner pay for each cloth hat?
(2) How many straw hats did the store-owner buy?

The store-owner sold each of the straw hats at a profit of 60% and each of the cloth hats at a profit of 50%.

- ב. (1) For how many shekels did the store-owner sell all 120 hats?
(2) What was the store-owner's percentage profit from the sale of all 120 hats?

2. Line AC passes through Points A(0, -6) and C(4, -4) (see figure).

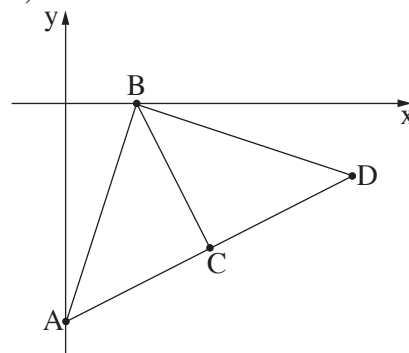
- א. (1) Find the slope of Line AC.
- (2) Find the equation of Line AC.

Line AB, whose equation is $y = 3x - 6$, intersects the x-axis at Point B.

- ב. Find the coordinates of Point B.

Point D is located on the extension of Segment AC. $AC = CD$.

- ג. Find the coordinates of Point D.
- ד. Prove that Triangle ABD is an isosceles triangle.
- ה. Find the area of Triangle ABD.



3. Given: A circle with center M(18, 12). Point A(9, 0) is located on the circle (see figure).

- א. (1) Find the length of the radius of the circle.
- (2) Write out the equation of the circle.

Point B in the figure is located on the circle, such that Segment BM is parallel to the x-axis (B is to the right of M).

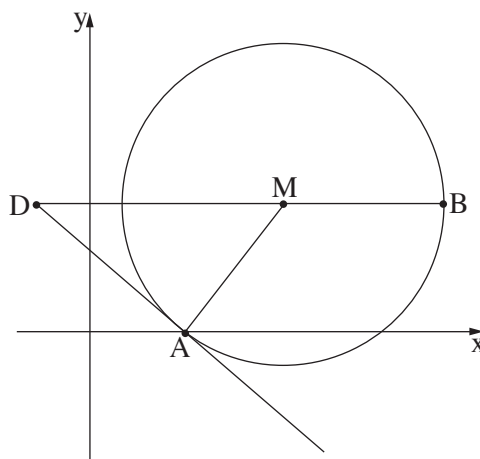
- ב. (1) Find the y-coordinate of Point B.
- (2) Find the x-coordinate of Point B.

A tangent to the circle was drawn through Point A.

- ג. (1) Find the slope of the tangent.
- (2) Find the equation of the tangent.

The tangent intersects the extension of Segment BM at Point D.

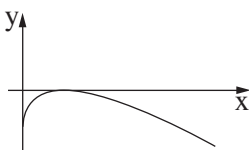
- ד. (1) Find the length of Segment DM.
- (2) Calculate the area of Triangle ADM.



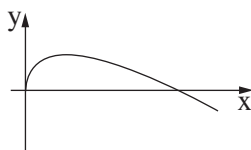
Differential and Integral Calculus

4. Given: the function $f(x) = -\frac{1}{6}x + \sqrt{x}$.

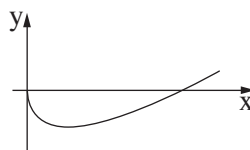
- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
- ב. Find the coordinates of the internal extremum [נקודת הקיצון הפנימית] of the function $f(x)$, and determine what type of extremum it is.
- ג. Find the coordinates of the point at which the graph of the function $f(x)$ intersects the y-axis.
- ד. Which of the four graphs (I-IV) at the end of the question is the graph of the function $f(x)$? Explain.
- ה. How many points of intersection does the line $y = -3$ have with the graph of the function $f(x)$? Explain.



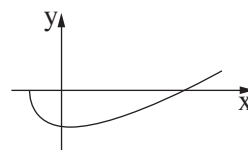
IV



III



II



I

5. The figure at the right shows the graph of the function $f(x) = 2x^3 + 2x^2 - 2x$.

Points A and B in the figure are the extrema [נקודות הקיצון] of the function $f(x)$.

- א. Find the coordinates of Points A and B.

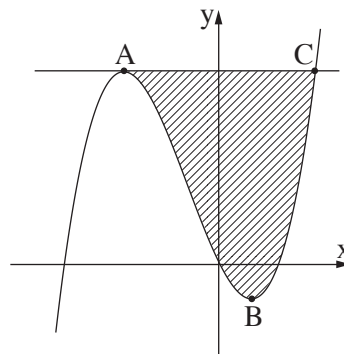
A tangent to the graph of the function was drawn through Point A.

- ב. Find the equation of the tangent.

The tangent to Point A also intersects the function at Point C.

The x-coordinate of Point C is 1.

- ג. Calculate the area of the region bounded by the graph of the function and the tangent (the shaded region in the figure).



6. Given: Rectangle ABCD, whose perimeter is 36 cm.

Let x denote the length of Side AB.

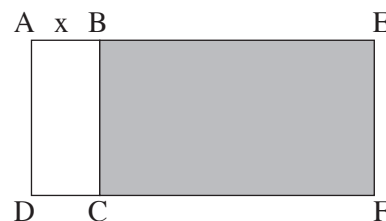
א. Express the length of Side BC in terms of x .

An additional rectangle, BEFC, is constructed on Side BC (the gray rectangle in the figure).

Side BE is 4 times longer than Side AB.

ב. (1) Express the area of Rectangle BEFC in terms of x .

(2) Find the value of x for which the area of Rectangle BEFC is maximal (greatest).



Good Luck!

All rights reserved by the State of Israel.
 Copying or publication without the consent of the
 Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך