The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2019
Exam Number: 035582
Appendix: Formula sheets for 5 points
English Translation (3)

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers
$2\times 33\frac{1}{3} - 66\frac{2}{3}$ points

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions
$1\times 33\frac{1}{3} - 33\frac{1}{3}$ points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשע"ט, 2019
מספר השאלון: 035582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים,
טריגונומטריה במרחב, מספרים מרוכבים
$2\times 33\frac{1}{3} - 66\frac{2}{3}$ נקודות

פרק שני: גדילה ודעיכה, פונקציות חזקה,
פונקציות מעריכיות ולוגריתמיות
$1\times 33\frac{1}{3} - 33\frac{1}{3}$ נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון,
גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים,
בפירוט ובצורה ברורה מסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers ($66\frac{2}{3}$ points)

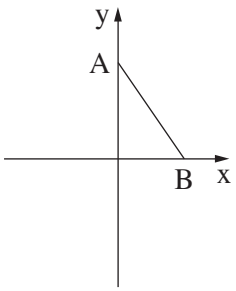
Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The length of the segment AB is 4.

Given: Point A is located on the y-axis, and Point B is located on the x-axis (see figure).

Point M is the midpoint of the segment AB .



א. Find the equation of the locus [המקום הגאומטרי] of all the points M that were constructed in this way, and identify this locus.

Given: Point L is located on the segment AB such that $\frac{AL}{LB} = t$. $t > 0$ is a parameter.

ב. Express in terms of t the equation of the locus of all the points L that were constructed in this way, and identify this locus.

ג. For what value of t does the locus you found in Item ב coincide with the locus you found in Item א? Explain.

ד. Is there a $t > 0$ for which the locus you found in Item ב intersects the x-axis at the point $(5, 0)$? Explain.

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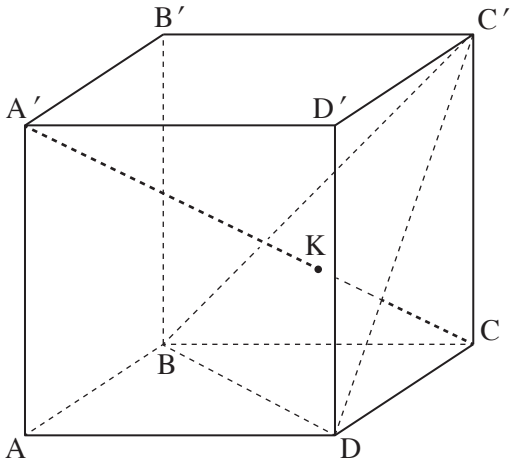
2. $ABCDA'B'C'D'$ is a cube with an edge length of 6 (see figure).

Point B is located at the origin.

א. Calculate the size of the angle between Segment $A'C$ and Segment BC'.

ב. Prove that the line $A'C$ is perpendicular to the plane $BC'D$.



Point K is the intersection point of the line $A'C$ with the plane $BC'D$.

ג. Find the ratio $\frac{A'K}{A'C}$.

Point O is the intersection point of the diagonal of the base AC with the diagonal of the base BD.

ד. Prove that Point K is located on the segment $C'O$.

3. **א.** **(1)** Prove that any complex number z satisfies $z \cdot \bar{z} = |z|^2$.

(2) Prove that if the complex number z is located on the unit circle [מעגל היחידה], then the number $\frac{1}{z}$ is also located on the unit circle.

ב. **(1)** Show that for any complex number z located on the unit circle, the sum $z + \frac{1}{z}$ is a real number [מספר ממשי].

(2) z_1 and z_2 are complex numbers located on the unit circle.

Given: The imaginary components of z_1 and z_2 are positive.

Prove that if $z_1 + \frac{1}{z_1} + z_2 + \frac{1}{z_2} > 2$, then z_1 and z_2 are in the first quadrant.

$w = 1 \cdot \text{cis}(\alpha)$ is a complex number. Given: $0 < \alpha < \frac{\pi}{2}$.

In a given geometric sequence, the first term is $\frac{1}{w}$ and the second term is w.

Given: The sum of the first 5 terms in the geometric sequence is 0.

ג. **(1)** Express the common ratio of this sequence in terms of α, and explain why all the terms of this sequence are located on the unit circle.

(2) Find α (find both possibilities).

Continued on page 4 ▶

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: the function $f(x)=\ell n\left(\frac{e^x}{e^x+1}\right)$, which is defined for all x.

א. **(1)** Find the coordinates of the points of intersection of the graph of the function f(x) with the axes (if there are any).

(2) Find the positive and negative intervals of the function f(x) (if there are any).

(3) Find the equation of the asymptote of the function f(x) that is parallel to the x-axis.

(4) Find the intervals on which the function f(x) is increasing and on which it is decreasing (if there are any).

ב. Sketch a graph of the function f(x).

ג. **(1)** Prove that $f(x)=x-\ell n\left(e^x+1\right)$.

(2) Explain why the graph of the function f(x) is located entirely under the line $y=x$.

ד. Given: The function $g(x)=\frac{1}{\sqrt{e^x+1}}$, which is defined for all x.

(1) What are the positive and the negative intervals of the function g(x) (if there are any)?

(2) $a>1$ is a parameter.

Use the derivative of the function f(x) to show that the volume of the solid of revolution [גוף הסיבוב] generated by the region bounded by the graph of the function g(x), the axes, and the line $x=\ell na$ equals $\pi\ell n\left(\frac{2a}{a+1}\right)$. Provide detailed calculations.

Continued on page 5 ▶

5. Given: the family of functions $f(x)=\frac{e^{-mx}}{1+x^2}$. m is a parameter.

Answer Items א and ב for $m \geq 0$.

א. **(1)** What is the domain [תחום ההגדרה] of the functions $f(x)$?

(2) Find the positive and negative intervals of the functions f(x) (if there are any). Explain.

(3) Given: All the functions f(x) of the family intersect at one point. Find the coordinates of this point.

ב. **(1)** For $m \geq 0$, find the values of m for which the derivative $f'(x)$:

(i) does not vanish [מתאפסת] at any point.

(ii) vanishes at exactly one point.

(iii) vanishes at exactly two points.

(2) Shown below are three graphs (I-III) of functions in the family of functions f(x) for $m \geq 0$.

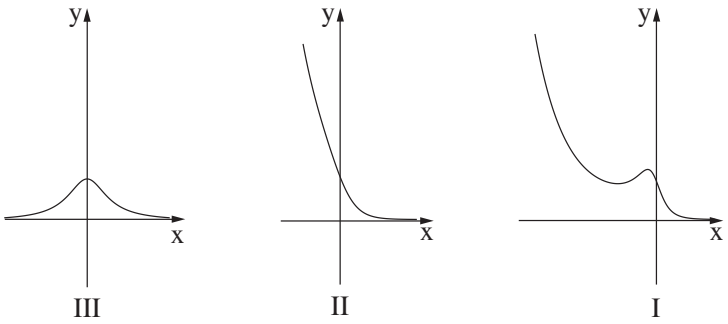
It is known that $m \neq 1$ and that each of the graphs corresponds to a different value or range of values of m.

Assign the correct value or range of values of m to each of the three graphs.

Explain.

Answer Item ג. You may make use of the appropriate graph from the given graphs I-III.

ג. Sketch a graph of the function $f(-x)$, for $0 < m < 1$.



Good Luck!

בהצלחה!