

Mathematics

4 Points — 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes

ב. Exam structure and breakdown of points:

This exam has two parts.

Part One — Sequences, Spatial Trigonometry

$$- 1 \times 33\frac{1}{3} - 33\frac{1}{3} \text{ points}$$

Part Two — Growth and Decay, Differential and Integral

Calculus of Trigonometric Functions,

Exponential and Logarithmic Functions, and

Power Functions

$$- 2 \times 33\frac{1}{3} - 66\frac{2}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

4 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון — סדרות, טריגונומטריה במרחב

$$- 33\frac{1}{3} \times 1 - 33\frac{1}{3} \text{ נקודות}$$

פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של

פונקציות טריגונומטריות,

פונקציות מעריכיות ולוגריתמיות

ופונקציות חזקה

$$- 33\frac{1}{3} \times 2 - 66\frac{2}{3} \text{ נקודות}$$

$$\text{סה"כ} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות

במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול

לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד, רשום "טיטה" בראש כל עמוד המשמש טיוטה. תחיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Sequences, Spatial Trigonometry ($33\frac{1}{3}$ points)

Answer one of the questions 1-2.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

Sequences

1. a_n is an arithmetic sequence whose first term is a_1 and whose common difference is 4.

b_n is a sequence that is defined by: $b_n = a_n + 8n$.

א. Prove that b_n is an arithmetic sequence and find its common difference.

c_n is a sequence that is defined by: $c_n = a_n + b_n$.

ב. Prove that c_n is an arithmetic sequence.

Given: $a_1 = \frac{1}{2}$.

ג. (1) Find the value of c_1 .

(2) Find the sum of the first 20 terms of the sequence c_n .

Spatial Trigonometry

2. $ABCA'B'C'$ is a right triangular prism whose base is an isosceles triangle ($AC = AB$).

Point D is the midpoint of Segment CB (see figure).

Given: $AD = 12$, $\angle CAB = 40^\circ$.

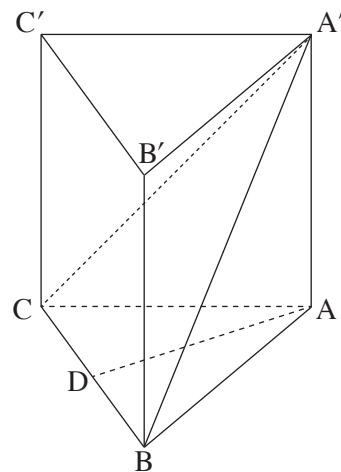
א. Calculate the length of Side CB.

ב. Explain why Triangle $CA'B$ is an isosceles triangle.

Given: The area of Triangle $CA'B$ is 80.

ג. Calculate the size of the angle between Segment DA' and the base of the prism, ABC .

ד. Calculate the volume of Prism $ABCA'B'C'$.



**Part Two — Growth and Decay, Differential and Integral Calculus of
Trigonometric Functions, Exponential and Logarithmic Functions,
and Power Functions** ($66\frac{2}{3}$ points)

Answer two of the questions 3-5 (each question — $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

3. The function $f(x)$ is defined on the domain $0 \leq x \leq \pi$.

Given: $f'(x) = -3 \sin 2x$, $f(0) = 0.75$.

The derivative function, $f'(x)$, is also defined on the domain $0 \leq x \leq \pi$.

- א. Find an algebraic expression for the function $f(x)$.
- ב. Find the coordinates of the points at which the graph of the function $f(x)$ intersects the x-axis.
- ג. Find the coordinates of all the extrema [נקודות הקיצון] of the function $f(x)$ for the given domain, and determine what kind of extrema they are.
- ד. Sketch a graph of the function $f(x)$.
- ה. Calculate the area of the region bounded by the graph of the function $f(x)$ and the x-axis over the interval between the points of intersection you found in Item ב.

4. Given: the function $f(x) = -3e^x(2e^x - 4)$.

- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
- ב. Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes.
- ג. Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$, and determine what kind of extremum it is.
- ד. Sketch a graph of the function $f(x)$.
- ה. Given: the function $g(x) = -\frac{1}{2}f(x)$.
 - (1) Write down the coordinates of the extremum of the function $g(x)$, and determine what kind of extremum it is.
 - (2) Sketch a graph of the function $g(x)$.

5. Given: the function $f(x) = \ln(-x^2 + ax)$, whose domain is $0 < x < a$. $a > 0$ is a parameter.

It is known that the function $f(x)$ has an extremum [נקודת קיצון].

א. Show that the x-coordinate of the extremum of the function $f(x)$ is $\frac{a}{2}$.

Given: The y-coordinate of the extremum of the function $f(x)$ is $\ln\left(2\frac{1}{4}\right)$.

ב. Find the value of a .

Substitute $a = 3$ in the equation of the function $f(x)$ and in its domain, and answer Items ג-ד.

ג. Determine the type of the extremum of the function $f(x)$.

ד. (1) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the x-axis. Your answer should be correct to two decimal places.

(2) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the x-axis.

(3) Sketch a graph of the function $f(x)$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך