

**The State of Israel
Ministry of Education**

Exam Type: *Bagrut*

Exam Date: Summer 2019

Exam Number: 035481

Appendix: Formula sheets for 4 points

English Translation (3)

מדינת ישראל

משרד החינוך

סוג הבחינה: בגרות

מועד הבחינה: קיץ תשע"ט, 2019

מספר השאלון: 035481

נספח: דפי נוסחאות ל-4 יחידות לימוד

תרגום לאנגלית (3)

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

Probability — 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus of

Polynomials, of Rational Functions, and of

Root Functions

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

- (2) Formula sheets (attached).

- (3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.

- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית,

הסתברות — 2×20 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

— 1×20 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

של פולינומים, של פונקציות רציונליות

ושל פונקציות שורש

— 2×20 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

- (2) דפי נוסחאות (מצורפים).

- (3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת

הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיטה" בראש כל עמוד המשמש טיטה.

כתיבת טיטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The distance between City α and City β is 120 km.

In the morning a car traveled from City α to City β at a constant speed.

That evening the car traveled back from City β to City α via the same route. The car traveled for one hour at the same speed at which it had traveled in the morning. It stopped on the side of the road for 2 minutes, and then continued the journey to City α at a speed that was 10 km/h greater than its speed in the morning.

The journey time in the evening (including the stop) was equal to the journey time in the morning.

α . Find the speed of the car in the morning.

- β . The car set off from City β on its way back to City α at eight o'clock in the evening.
How far was the car from City α at 8 minutes past nine in the evening?

2. A circle is given whose center is $M(7, 6)$. Line MB intersects the circle at Point C (see figure).

Given: $B(1, 14)$,

$$MC = CB.$$

- א. Find the equation of the circle.

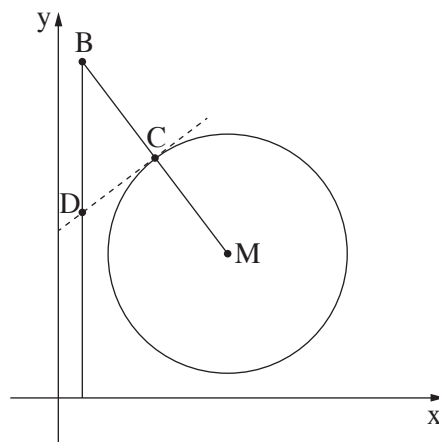
A tangent to the circle was drawn through Point C .

- ב. Find the equation of this tangent.

A perpendicular was drawn from Point B to the x -axis.

The tangent and perpendicular intersect at Point D .

- ג. Calculate the area of Triangle BCD .



Point E is located on the perpendicular that was drawn from Point B to the x -axis.

Given: $ME \parallel CD$.

- ד. Find the coordinates of Point E .

- ה. Show that Point D is the center of the circle that circumscribes [חוסם] Triangle BME .

3. A game consists of two rounds. In each round there are only two possibilities: winning or losing. A player who wins both rounds wins the whole game.

The probability of winning the first round is 3 times greater than the probability of losing the first round.

- א. What is the probability of winning the first round? Explain.

If a player wins the first round, the probability that he/she will win the second round is 0.8.

If a player loses the first round, the probability that he/she will win the second round is 0.6.

- ב. (1) What is the probability of winning precisely one of the two rounds?
(2) We know that a player won precisely one of the two rounds. What is the probability that he/she won the first round?
- ג. (1) What is the probability of winning the whole game?
(2) 4 people are playing this game. What is the probability that all the players will win the whole game?

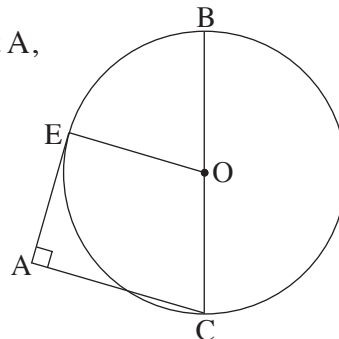
Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: a circle whose center is O.

BC is a diameter of the circle. Two lines were drawn from Point A, which is located outside the circle: One line is tangent to the circle at Point E, and the other intersects the circle at Point C, as shown in the figure on the right.



Given: $\angle EAC = 90^\circ$.

- א. Prove that $EO \parallel AC$.
- ב. Prove that $\angle OCE = \angle ACE$.
- ג. Prove that $\triangle EBC \sim \triangle AEC$.

Given: $BC \cdot AC = 64$.

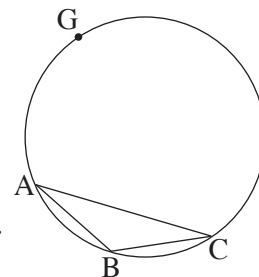
- ד. (1) Calculate EC.
- (2) Given: $EB = 6$.
Calculate EO.

5. An isosceles triangle ABC ($AB = BC$) is inscribed [חסום] in a circle with a radius of 10, as shown in the figure on the right.

Given: $\angle ABC = 130^\circ$.

- א. Calculate the length of Side AC.
- ב. Calculate the area of Triangle ABC.

G is a point located on the circle, such that GC is a diameter of the circle. Line GB intersects Side AC at Point E.



- ג. Calculate the length of Segment EB.

Part Three — Differential and Integral Calculus of Polynomials, of Rational Functions, and of Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = \frac{3x^2}{x^2 + x - 2}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the asymptotes of function $f(x)$ that are parallel to the axes.
- (3) Find the coordinates of the point at which the graph of function $f(x)$ intersects the axes.
- (4) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
- (5) Find the intervals on which function $f(x)$ is increasing and decreasing.
- ב. Sketch the graph of function $f(x)$.
- ג. Does the graph of function $f(x)$ intersect the horizontal asymptote of function $f(x)$? If it does intersect the asymptote, find the coordinates of the point of intersection.
- ד. Given: The function $g(x) = f(x) + c$ (c is a parameter) has a horizontal asymptote $y = 5$. Find the value of c . Explain.

7. Given: the function $f(x) = -\frac{1}{3}x^3 + 9x + a$, which is defined for all x . a is a parameter.

- א. Find the coordinates of the point at which the graph of function $f(x)$ intersects the y -axis (if necessary, express in terms of a).
- ב. Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$ (if necessary, express in terms of a), and determine what type of extrema they are.
- ג. Find the value of a for which the minimum point of function $f(x)$ is located on the x -axis. Explain.

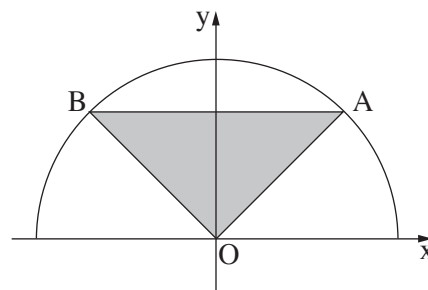
Substitute $a = 18$ in the equation of function $f(x)$ and answer Items ו-ד.

- ד. Give the coordinates of the extrema of the function.
- ה. Sketch the graph of function $f(x)$.
- ו. (1) Calculate the area in the second quadrant that is bounded by the graph of function $f(x)$, the x -axis, and the y -axis.
- (2) A is the point of intersection of the graph of function $f(x)$ with the y -axis, and B is the minimum point of function $f(x)$. Show that the graph of function $f(x)$ divides Triangle ABO into two areas whose ratio is 1:3 (O is the origin).

8. The figure on the right shows the graph of the function $f(x) = \sqrt{25 - x^2}$, which is defined on the domain $-5 \leq x \leq 5$.

Point A is located on the graph of function $f(x)$ in the first quadrant.

A line parallel to the x-axis was drawn through Point A. The line intersects the graph of function $f(x)$ at Point B, which is in the second quadrant. O is the origin.



Let t denote the x-coordinate of Point A.

- א. (1) Express the coordinates of Point B in terms of t .
- (2) Express the area of Triangle ABO in terms of t .
- ב. Find the value of t for which the area of Triangle ABO is maximal (largest).
You may leave a root sign in your answer.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך