

Mathematics

3 Points — 3rd Exam

Instructions for examinees

א. Duration of the exam: Two hours

ב. Exam structure and breakdown of points:

This exam has six questions in the topic areas:

Algebra, Differential and Integral Calculus.

You must answer four questions — $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

- (2) Formula sheets (attached)

- (3) A Hebrew-foreign language/
foreign language-Hebrew dictionary

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of your exam.

Write in the answer booklet only. Write "טיטה" at the top of every page you use for rough work.

If you use pages outside the answer booklet for rough work, your exam may be disqualified.

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שעותיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות — $4 \times 25 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

- (2) דפי נוסחאות (מצורפים).

- (3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.

- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיטה" בראש כל עמוד המשמש טיטה.

כתיבת טיטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-6 (each question – 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. A certain swimming pool sells entry punch-cards [כרטיסיות] for adults and entry punch-cards for children.

The price of an adults' punch-card is 1.6 times greater than the price of a children's punch-card.

Toward the end of the swimming season, punch-cards were sold at a discount. After the discount, the price of an adults' punch-card was 20% less than the original price, and the price of a children's punch-card was 10% less than the original price.

Dana bought one adults' punch-card and 4 children's punch-cards at the discounted price. She paid a total of 854 shekels for all the punch-cards she bought.

- א. (1) Find the original price of a children's punch-card (without the discount).
(2) Find the original price of an adults' punch-card (without the discount).

At the beginning of the swimming season, Shir bought punch-cards at the original price (without the discount). She too bought one adults' punch-card and 4 children's punch-cards.

- ב. (1) How much did Shir pay in total for all the punch-cards she bought?
(2) By what percentage was the total sum that Shir paid for her punch-cards greater than the total sum that Dana paid for her punch-cards?

Your answer should be correct to two decimal places.

2. ADC is a right triangle ($\angle ADC = 90^\circ$).

Point A lies on the y-axis.

Point B is the point of intersection of Line AD and the x-axis (see figure).

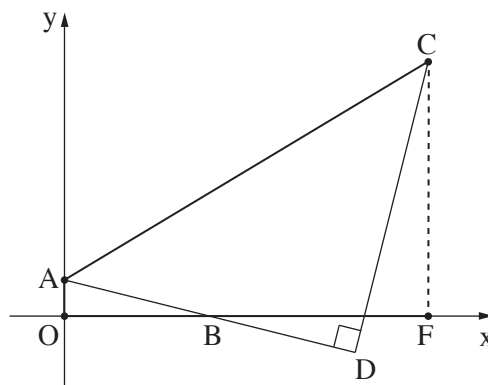
Given: The equation of Line AD is $y = -\frac{1}{4}x + 1$.

א. Find the coordinates of Points A and B.

Given: Point B is the midpoint of Segment AD.

א. (1) Find the coordinates of Point D.

(2) Find the equation of Line DC.



A perpendicular is dropped from Point C to the x-axis. This perpendicular intersects the x-axis at Point F.

Given: The x-coordinate of Point C is 10.

Point O is the origin.

א. Calculate the perimeter of Quadrilateral OACF.

Your answer should be correct to two decimal places.

3. The figure to the right shows a circle whose center is Point M.

Point B is located on the circle.

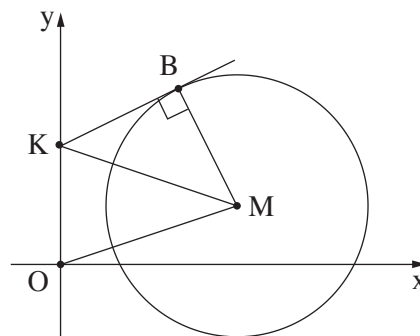
The equation of the tangent to the circle at Point B is $y = \frac{1}{2}x + 4$.

The x-coordinate of Point B is 4.

א. (1) Find the y-coordinate of Point B.

(2) Find the slope of Line BM.

(3) Find the equation of Line BM.



The equation of Line OM is $y = \frac{1}{3}x$ (O is the origin).

א. (1) Find the coordinates of Point M.

(2) Find the equation of the circle.

The tangent to the circle at Point B intersects the y-axis at Point K (see figure).

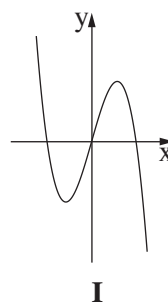
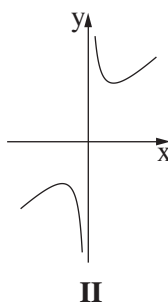
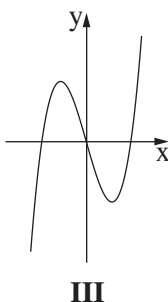
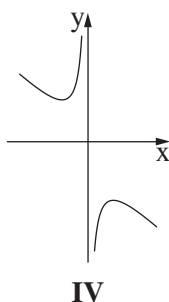
א. (1) Find the coordinates of Point K.

(2) Calculate the area of Triangle BMK.

Differential and Integral Calculus

4. Given: function $f(x) = x + \frac{9}{x} + 1$.

- א. (1) What is the domain [תחום ההגדרה] of function $f(x)$?
- (2) Write out the equation of the vertical asymptote of function $f(x)$.
- ב. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ג. Find the intervals on which the function is increasing and decreasing.
- ד. Which of the graphs I-IV at the end of the question is the graph of the given function $f(x)$? Explain.
- ה. How many points of intersection does the line $y = 9$ have with the graph of the function? Explain.



5. Given: function $f(x) = -x^2 + 6x - 5$.

Points A and B are the points of intersection of the graph of function $f(x)$ and the x-axis, as shown in the figure.

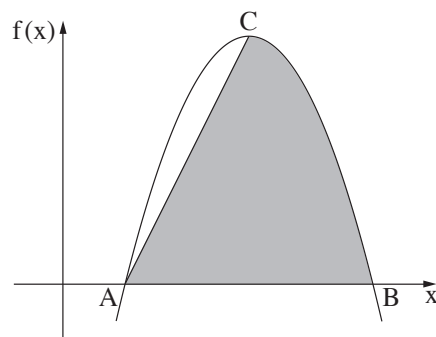
- א. Find the coordinates of Points A and B.

Point C is the maximum point of function $f(x)$.

- ב. Find the coordinates of Point C.

- ג. Show that the equation of Line AC is $y = 2x - 2$.

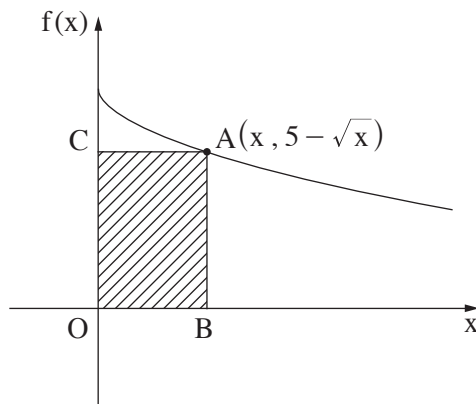
- ד. Calculate the area of the shaded region in the figure: the region bounded by the graph of function $f(x)$, Line AC, and the x-axis.



6. Given: function $f(x) = 5 - \sqrt{x}$.

Perpendiculars were dropped to the axes from Point A, which is located on the graph of function $f(x)$ in the first quadrant, thereby forming Rectangle ABOC, as shown in the figure (Point O is the origin).

- א. (1) Express the perimeter of Rectangle ABOC in terms of x .
(2) Find the x -coordinate of Point A for which the perimeter of Rectangle ABOC is minimal (smallest).
- ב. Find the perimeter of Rectangle ABOC for the x -coordinate you found in Sub-item א(2).



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך