The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2019
Exam Number: 035582
Appendix: Formula sheets for 5 points
English Translation (3)

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers
$2\times 33\frac{1}{3} - 66\frac{2}{3}$ points

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions
$1\times 33\frac{1}{3} - 33\frac{1}{3}$ points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

(3) For draft paper, use the pages in the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשע"ט, 2019
מספר השאלון: 035582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים,
טריגונומטריה במרחב, מספרים מרוכבים
$2\times 33\frac{1}{3} - 66\frac{2}{3}$ נקודות

פרק שני: גדילה ודעיכה, פונקציות חזקה,
פונקציות מעריכיות ולוגריתמיות
$1\times 33\frac{1}{3} - 33\frac{1}{3}$ נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון,
גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים,
בפירוט ובצורה ברורה מסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטיוטה יש להשתמש במחברת הבחינה.
שימוש בטיוטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

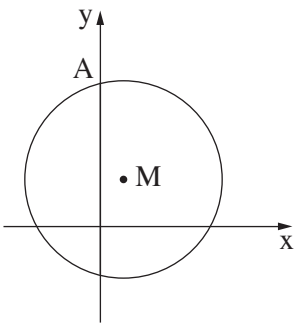
Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. A circle centered at Point M intersects the positive section of the y-axis at Point A, as shown in the figure at the right.



A line was drawn from the center of the circle, perpendicular to the y-axis and intersecting the y-axis at Point E.

Given: AE = 6 .

Given also: The distance of Point M from the origin is half of the length of the radius of the circle.

א. Prove that the locus [מקום גאומטרי] of all the points M that satisfy the data given in this question is located on an ellipse, and find the equation of that ellipse.

Let F_1 and F_2 denote the foci of the ellipse [מוקדי האליפסה] whose equation you found in Item א.

Points D_1 and D_2 are points on the ellipse.

The y-coordinate of D_1 is positive, and the y-coordinate of D_2 is negative.

ב. **(1)** Find the greatest possible area for the quadrilateral $F_1D_1F_2D_2$. Explain.

(2) Is there a quadrilateral $F_1D_1F_2D_2$ whose perimeter is greatest? Explain.

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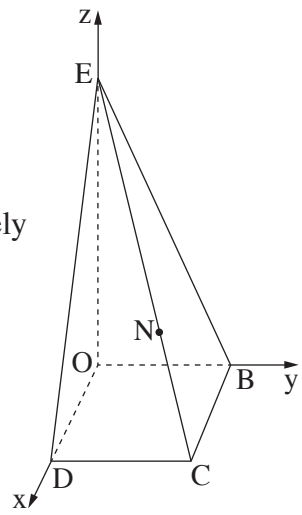
2. Given: Pyramid OBCDE, whose base OBCD is a square.
Edge OE is perpendicular to the plane of the base.
Given: OD = 4, OE = 12.
The pyramid was placed on a set of axes so that Point O is the origin and Points D, B, and E are located on the x-, y-, and z-axes, respectively (as shown in the figure).



א. Find the parametric representation of Line EC.

Point N is located on Edge EC. A perpendicular to the base of the pyramid is dropped from Point N, intersecting the base at Point F.
The distance of Point F from the y-axis is 3.

ב. Find the coordinates of Point N.

ג. Find the size of the angle between Plane BCN and the base of the pyramid, OBCD.

K is a point on Edge EC such that KOBCD is a right pyramid (OBCD is the pyramid's base).

ד. Find the parametric representation of the line on which the height to the base from Point K lies.

3. **א**. Find all the solutions of the equation $z^3 = \bar{z}$, $z \neq 0$. z is a complex number.

ב. **(1)** Find the equation of the locus [מקום גאומטרי] of all points in the Gaussian plane that satisfy $z^2 \cdot (\bar{z})^2 = 1$.

(2) Show that all solutions of the equation in Item א are located on the locus you found in Sub-item ב(1).

ג. **(1)** The points corresponding to the equation's solutions that you found in Item א are rotated by a 45° angle (counter-clockwise).
Given: Each of the points obtained following the rotation corresponds to a number that satisfies the equation $z^4 = a$.
Find the value of a .

(2) The points corresponding to the equation's solutions that you found in Item א are rotated by an angle of size α (counter-clockwise).
Prove that the sum of the numbers corresponding to the points obtained following the rotation is equal to 0.

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Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: the function $f(x)=\dfrac{e^{ax}-e^{x}}{e^{ax}-3e^{x}+2}$, a is a parameter.

The function has a vertical asymptote $x=\ell n2$.

א. Find the value of a.

Substitute the value of a that you found, and answer Items ב-ה.

ב. Find the domain [תחום ההגדרה] of the function f(x) and show that for all $x\neq 0$ in the domain, $f(x)=\dfrac{e^{x}}{e^{x}-2}$ is satisfied.

ג. **(1)** Find the asymptotes of the function f(x) that are perpendicular to the axes.

(2) Find the intervals on which the function f(x) is increasing and the intervals on which it is decreasing (if there are such intervals).

(3) Sketch a graph of the function f(x).

We define a function $h(x)=\left|\dfrac{e^{x}}{e^{x}-2}-\dfrac{1}{2}\right|$.

ד. **(1)** List the asymptotes of the function h(x) that are perpendicular to the axes.

(2) Sketch a graph of the function h(x).

(3) Calculate the area of the region bounded by the graph of the function h(x), by the x-axis, and by the lines $x=\ell n8$ and $x=\ell n16$.

Given: The function h(x) is symmetric with respect to the line $x=\ell n2$.

Points A and B are located on the graph of the function h(x), and they are symmetric with respect to the line $x=\ell n2$.

The x-coordinate of Point A is $\ell n8$.

ה. Find the coordinates of Point B.

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5. f(x) is a function that is differentiable [גזירה] for all values of x in its domain [תחום ההגדרה].

א. Show that the function f(x) and the function $e^{f(x)}$ have extrema [נקודות קיצון] at the same x-coordinates, and that these extrema are of the same type (minimum or maximum).

Given: $f(x) = x\ell n(x^n)$. n is a parameter that is a natural number.

ב. Find the domain of the function f(x). Consider different values of n .

ג. Find the coordinates of the points of intersection of the graph of f(x) with the x-axis. Consider different values of n .

Given: The graph of the function f(x) intersects the x-axis at two points.

ד. **(1)** Show that the function f(x) is an odd function.

(2) Find the coordinates of the extrema of the function f(x), and determine what type of extrema they are (if necessary, express in terms of n).

(3) Sketch a graph of the function f(x) for $n = 2$.

ה. Using your answer to Item א, find the coordinates of the extrema of the function $e^{x\ell n(x^n)}$ (n is a natural number), and determine what kind of extrema they are (if necessary, express in terms of n).

Good Luck!

בהצלחה!