

Mathematics

4 Points — 1st Exam

Instructions for examinees

- א. Duration of the exam: Three and a half hours
- ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra, Analytical Geometry,
Probability — 2×20 — 40 points
Part Two: Planar Geometry and Planar Trigonometry
— 1×20 — 20 points
Part Three: Differential and Integral Calculus of
Polynomials, Rational Functions, and
Root Functions
— 2×20 — 40 points
Total — 100 points
- ג. Material that may be used during the exam:
 - (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
 - (2) Formula sheets (attached).
 - (3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.
- ד. Special instructions:
 - (1) Do not copy the question, but do indicate the number of the question you are answering.
 - (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
 - (3) For draft paper, use pages in the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

- א. משך הבחינה: שלוש שעות וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות
 2×20 — 40 נקודות
פרק שני: גאומטריה וטריגונומטריה במישור
 1×20 — 20 נקודות
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים,
של פונקציות רציונליות ושל פונקציות שורש
 2×20 — 40 נקודות
סה"כ — 100 נקודות
- ג. חומר עזר מותר בשימוש:
 - (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
 - (2) דפי נוסחאות (מצורפים).
 - (3) מילון עברי-לועזי/לועזי-עברי.
- ד. הוראות מיוחדות:
 - (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
 - (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
 - הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
 - חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
 - (3) לטייטה יש להשתמש במחברת הבחינה.
 - שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. In Pizzeria Napoli, a family pizza costs 3 times more than a personal pizza.

The pizzeria announced a special offer:

a 10% discount on personal pizzas,

a 20% discount on family pizzas.

All the 11th grade students got together and bought 63 pizzas at the special price, some of them personal pizzas and some family pizzas.

Given: The number of family pizzas was 2.5 times greater than the number of personal pizzas.

The students paid a total of 3,477.6 shekels for all the pizzas.

- א. Calculate the original price of a personal pizza and the original price of a family pizza (before the discount).

- ב. A week later another special offer was announced:

If you pay the original price for two personal pizzas, you get the third personal pizza free.

How many personal pizzas can you buy for 1,232 shekels at the special rate (including the free pizzas)?

2. A circle with center M is given in the figure below.

A line passing through the origin is tangent to the circle at point $B(3, 4)$.

The center of the circle, M , was connected to the origin, O .

Given: The equation of the line OM is $y = \frac{1}{7}x$.

א. Find the equation of the line BM .

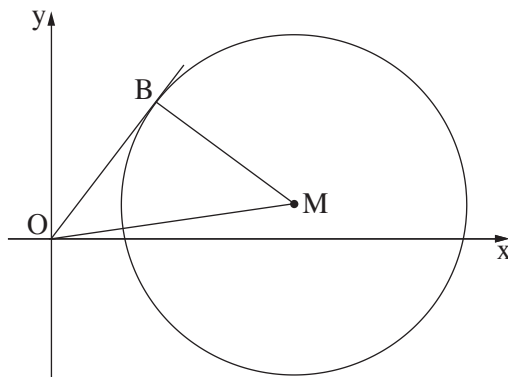
ב. Find the equation of the circle.

The extension of the segment BM intersects the circle at Point C .

ג. Find the area of Triangle OBC .

Another circle is drawn such that OM is a diameter of the new circle.

ד. Does the center of the second circle lie inside, on, or outside the circle whose center is M ? Provide detailed calculations and explain them.



3. Exactly 8% of the members of a national judo club have a black belt.

א. 6 club members are selected at random.

(1) What is the probability that exactly 2 of them have a black belt?

(2) What is the probability that none of the 6 selected members have a black belt?

$\frac{1}{5}$ of the club members are instructors and the rest are trainees.

75% of the club members with a black belt are instructors.

ב. A club member is selected at random.

What is the probability that the selected member is a trainee with a black belt?

ג. A trainee club member is selected at random.

What is the probability that he or she has a black belt?

Part Two — Planar Geometry and Planar Trigonometry (20 points)

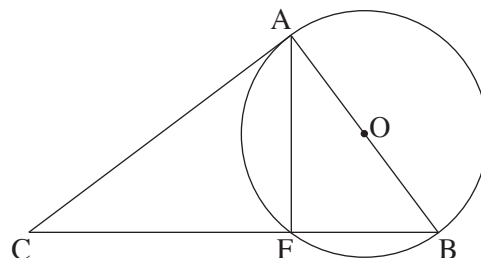
Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: a circle with a center O .

C is a point outside the circle, such that the line CA is tangent to the circle at Point A .

A line that intersects the circle at Points F and B is drawn from Point C , as shown in the figure, such that AB is a diameter of the circle.



⌘. Prove that: $\triangle AFB \sim \triangle CAB$.

Given: $FB = 9$, $FC = 16$.

Ⓐ. Calculate the diameter of the circle, AB .

Ⓐ. Calculate the area of the triangle CFA .

Ⓐ. Is $\triangle CFA \sim \triangle CAB$? Prove your answer.

5. Given: Triangle ABC .

Point D is located on the side AB such that $BD = 2DA$ (see figure).

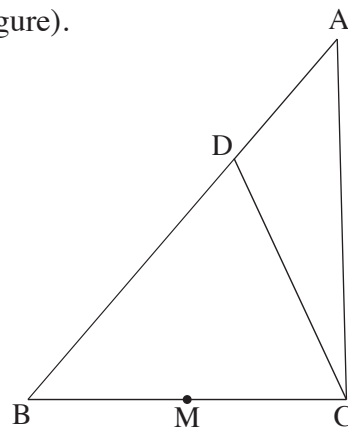
Given: $\angle DCB = 65^\circ$, $DC = 10$, $BC = 12$.

⌘. Calculate the length of the segment BD .

Ⓐ. Calculate the area of the triangle ADC .

Point M is the midpoint of the segment BC .

Ⓐ. Is Point M the center of the circle that circumscribes the triangle BDC ? Explain.



Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = -2 + \sqrt{-x^2 + 5x}$.

- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
- ב. What are the coordinates of the intersection points of the graph of the function $f(x)$ with the x -axis?
- ג. Find the coordinates of all the extrema [נקודות הקיצון] of the function $f(x)$ and determine what type of extrema they are.
- ד. On which intervals is the function $f(x)$ increasing and on which intervals is it decreasing?
- ה. Sketch a graph of the function $f(x)$.

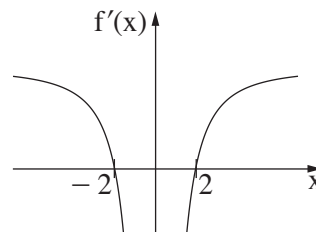
Given: the function $g(x) = f(x) + c$, whose domain is the same domain as of the function $f(x)$.
 c is a parameter.

- ו. What are all the values of c for which the function $g(x)$ is positive over its entire domain?

7. Function $f(x)$ is defined for all $x \neq 0$.

The figure below describes the graph of the derivative function $f'(x)$, which is also defined for all $x \neq 0$, and which intersects the x -axis at the points $(2, 0)$, $(-2, 0)$.

- א. Find the x -coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and use the graph to determine what type of extrema they are.



Given: $f'(x) = -\frac{1}{x^2} + a$ for all $x \neq 0$. $a > 0$ is a parameter.

- ב. Find the value of a .

Answer Item א for $x > 0$.

The y -coordinate of the minimum point of the function $f(x)$ is 10.

- ג. (1) Provide an algebraic expression for the function $f(x)$.
- (2) Sketch a graph of the function $f(x)$ for $x > 0$.

8. Rectangle DFGE is inscribed in the graph of the parabola $y = -x^2 + 6x$ and the x-axis, as shown in the figure.

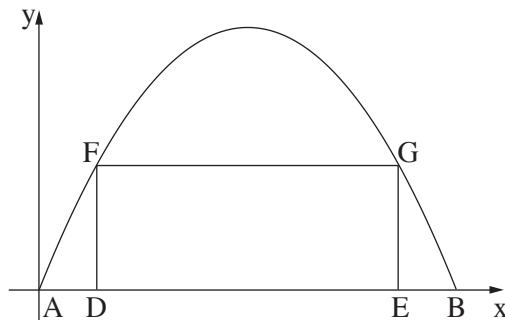
Points A and B are the parabola's intersection points with the x-axis, as shown in the figure.

k is a parameter. Given: $0 < k < 3$.

Given: $AD = EB = k$.

- א. Express the lengths of the sides of the rectangle DFGE in terms of k .

- ב. Find the value of k for which the area of the rectangle DFGE is greatest [מקסימלי]. You may leave a root sign in your answer.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך