

Mathematics

3 Points — 3rd Exam

Instructions for examinees

א. Duration of the exam: Two hours.

ב. Exam structure and breakdown of points:

This exam has six questions in the topic areas:

Algebra, Differential and Integral Calculus.

You must answer four questions — $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of your exam.
- (3) For draft paper, use the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שעותיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות — $4 \times 25 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
- (3) לטייטה יש להשתמש במחברת הבחינה. שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-6 (each question – 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. At the beginning of the year, a retailer bought some shirts and paid the same price for each shirt. He paid a total of 2,040 shekels for all the shirts together.

5 shirts had defects, so the retailer sold them at a loss of 10% per shirt.

The rest of the shirts were sold at a profit of 20% per shirt.

The retailer sold all the shirts for 2,412 shekels in total.

א. Find the price that the retailer paid for each shirt.

The retailer found in his storehouse 15 more shirts that he had bought the previous year, and sold them at a profit of 10% per shirt. (The price he had paid for a shirt the previous year was the same as the price he paid per shirt at the beginning of the current year.)

ב. (1) How much did the retailer pay in total for all the shirts that he sold?

(2) What was the retailer's overall percentage profit from the sale of all the shirts?

2. In Triangle ABC in the figure at the right,
given: A(9, 24), and B(1, 0).

א. Find the equation of Line AB.

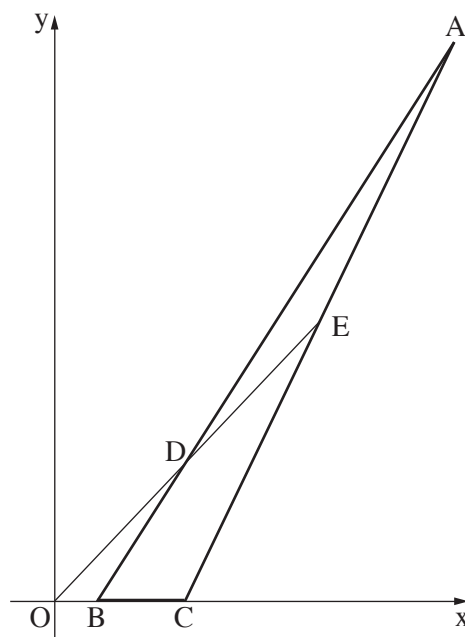
Line OE, whose equation is $y = 2x$, intersects Sides AB and AC at Points D and E, respectively (O is the origin).

ב. Find the coordinates of Point D.

Given: Vertex C lies on the x-axis, and

Point E is the midpoint of Segment AC.

- ג. (1) Find the y-coordinate of Point E.
(2) Find the x-coordinate of Point E.
ד. (1) Explain why Line DC is parallel to the y-axis.
(2) Calculate the perimeter of Triangle BCD.



3. Given: a circle whose center is M and whose equation is: $(x - 4)^2 + (y - 3)^2 = 10$.

Points A and B are the intersection points of the circle with the x-axis, as shown in the figure.

א. Find the coordinates of Points A and B.

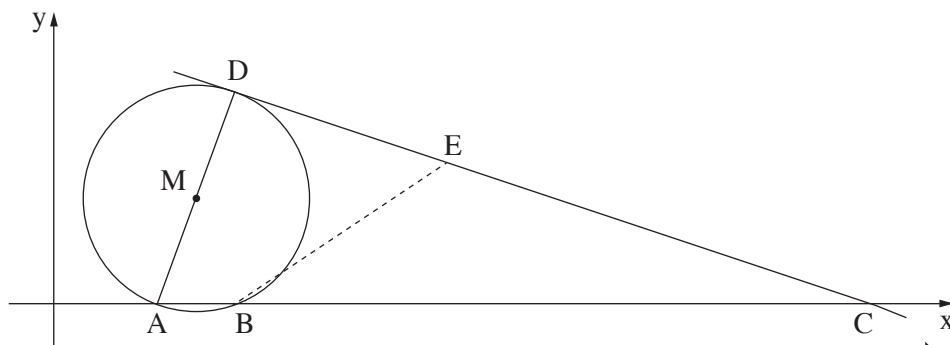
Point D is located on the circle such that AD is a diameter of the circle.

- ב. Find the coordinates of Point D.
ג. Find the equation of the tangent to the circle at Point D.

The tangent whose equation you found in Item ג intersects the x-axis at Point C.

Point E is located on the tangent and its x-coordinate is 11.

- ד. (1) Find the y-coordinate of Point E.
(2) Calculate the area of Triangle BEC.



Differential and Integral Calculus

4. Given: the function $f(x) = 12\sqrt{x} - 3x$.

- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
- ב. Find the coordinates of the point of intersection of the graph of the function $f(x)$ with the y-axis.
- ג. Find the coordinates of the internal extremum [נקודת הקיצון הפנימית] of the function $f(x)$, and determine what type of extremum it is.
- ד. Find the interval on which the function $f(x)$ is increasing and the interval on which it is decreasing.

5. Given: the function $f(x) = -2x^2 + 16x - 14$.

Points A and B are the points of intersection of the graph of the function $f(x)$ with the x-axis, as shown in the figure at the right.

Point C is the extremum [נקודת הקיצון] of the function $f(x)$.

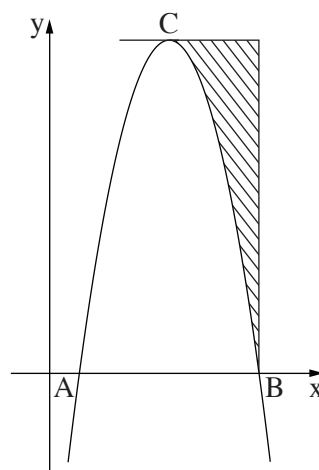
- א. Find the coordinates of Points A and B.
- ב. Find the coordinates of Point C.

A tangent was drawn to the function $f(x)$ at Point C.

- ג. Find the equation of the tangent.

A perpendicular to the x-axis was drawn from Point B.

- ד. Calculate the area of the shaded region in the figure: the region bounded by the graph of the function $f(x)$, the tangent, and the perpendicular.



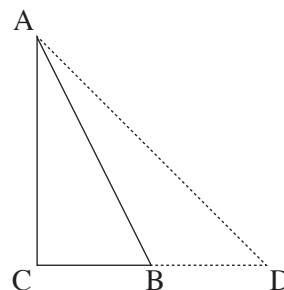
6. Triangle ABC is a right triangle ($\angle C = 90^\circ$).

Given: The area of Triangle ABC is 16.

Let x denote the length of Side CB.

- א. Express the length of Side AC in terms of x .

Side CB was extended by x , forming a new triangle, ACD, as shown in the figure at the right.



- ב. Find the value of x for which **the sum of the legs AC and CD** in the new triangle ACD is minimal (smallest).

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך