

Mathematics

5 Points – 2nd Exam

Instructions for examinees

א. Duration of the exam: Two hours and fifteen minutes

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors,

Spatial Trigonometry, Complex Numbers

$$- 2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ points}$$

Part Two: Growth and Decay, Power Functions,

Exponential and Logarithmic Functions

$$- 1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

(1) A non-graphing calculator.

You may not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

(3) For draft paper, use the pages in the answer booklet.

Use of other draft paper may lead to the disqualification of your exam.

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שתיים ורבע

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

$$- 66 \frac{2}{3} - 33 \frac{1}{3} \times 2 \text{ נקודות}$$

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

$$- 33 \frac{1}{3} - 33 \frac{1}{3} \times 1 \text{ נקודות}$$

$$\text{סה"כ} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול

לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטיטה יש להשתמש במחברת הבחינה.

שימוש בטיטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Given: the canonical parabola [פרבולה קנונית] $y^2 = 2px$. $p > 0$ is a parameter.

The points $A(x_1, y_1)$ and $B(x_2, y_2)$ are located on the parabola.

Given: The slope of the line AB is $\frac{4}{3}$,

the y-coordinate of the midpoint of the segment AB is 9.

- א. Find the equation of the parabola.

Given: The tangents to the parabola at Points A and B are perpendicular to each other.

- ב. Find the coordinates of Points A and B (Point A is located in the first quadrant).
ג. Find another pair of points on the parabola at which the tangents to the parabola are perpendicular to each other.

2. In Cuboid ABCDA'B'C'D', Point L is the midpoint of Edge BB', and Point G is the intersection point of the diagonals of Face A'B'C'D'.

Point K is the midpoint of Segment LG (see figure).

Let: $\overrightarrow{AD} = \underline{u}$, $\overrightarrow{AB} = \underline{v}$, $\overrightarrow{AA'} = \underline{w}$.

א. Express $\overrightarrow{DK}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

- ב. Prove that Point K is located on Diagonal DB', and find the ratio $\frac{DK}{DB'}$.

The extension of Segment AK intersects Plane BCC'B' at Point F.

Given: $\overrightarrow{AF} = s \cdot \underline{u} + \underline{v} + t \cdot \underline{w}$.

- ג. (1) Find s and t , and show that Point F is located on Edge B'C'.
(2) Find the ratio $\frac{B'F}{B'C'}$.

3. z_A , z_B , and z_C are three different complex numbers that represent Points A, B, and C, respectively, on the Gauss plane.

Given: $|z_A| = |z_B| = |z_C| = \sqrt{65}$,

Point A is located in the first quadrant,

z_A and z_C satisfy the equation: $(8 - i)z = (8 + i)\bar{z}$.

- א. (1) Find z_A and z_C .
(2) Explain why $\angle ABC = 90^\circ$.

Given: $AB = BC$.

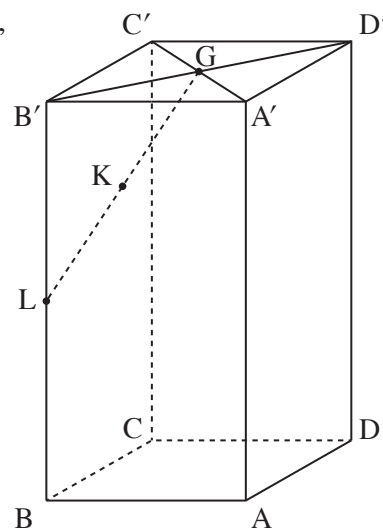
- ב. Find z_B (find both possibilities).

Given: Point B is located in the second quadrant.

- ג. a_n is a geometric sequence in which $a_1 = z_A$ and $a_2 = z_B$.

Given: m is a natural number such that the sum of the first m terms of the sequence a_n is 0.

Explain why m is divisible by 4 (without a remainder).



Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: the function $f(x) = \frac{e^x - 1}{e^x - x}$.

Let $g(x) = e^x - x$.

א. (1) What is the domain [תחום ההגדרה] of the function $g(x)$?

(2) Find the coordinates of the extremum [נקודת קיצון] of the function $g(x)$, and explain why $e^x - x \geq 1$ is satisfied for every value of x .

ב. (1) What is the domain of the function $f(x)$? Explain.

(2) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes (if there are any).

(3) Find the coordinates of the points of intersection of the graph of the function $f(x)$ with the axes (if there are such points).

(4) Show that $f'(x) = \frac{2e^x - xe^x - 1}{(e^x - x)^2}$.

We know that the expression $2e^x - xe^x - 1$ is defined for every x and is positive over the interval $-1 \leq x \leq 1$.

ג. (1) Calculate $f(-1)$ and $f(1)$, and sketch the graph of the function $f(x)$ over the interval $-1 \leq x \leq 1$.

(2) Use the preceding items of the question to explain why the function $f(x)$ has at least two extrema over its entire domain.

ד. Calculate the area bounded by the x -axis, the line $x = -1$, and the graph of the function $f(x)$ over the interval $-1 \leq x \leq 0$.

5. Given: the function $f(x) = \ln(e^{2x} + b)$. $b > 0$ is a parameter.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
(2) Find the intervals over which the function $f(x)$ is increasing and decreasing (if there are such intervals).

Given: the function $g(x) = \ln(e^x + be^{-x})$.

- ב. Find the domain of the function $g(x)$.
ג. (1) Prove: $f(x) - g(x) = x$.
(2) Find the coordinates of the intersection point of the graphs of the functions $f(x)$ and $g(x)$ (express in terms of b if necessary).

Given: The minimum point of function $g(x)$ is located on the asymptote of function $f(x)$.

- ד. Find the value of the parameter b .
ה. Substitute $b = 4$ and sketch the graphs of the functions $f(x)$ and $g(x)$ on a single set of axes.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך