The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2018, ***Moed Bet***
Exam Number: 035581
Appendix: Formula sheets for 5 points
English Translation (3)

Mathematics
5 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts.

Part One –	Algebra and Probability	
	– 2 × 20 –	40 points
Part Two –	Planar Geometry and Planar Trigonometry	
	– 1 × 20 –	20 points
Part Three –	Differential and Integral Calculus of Polynomials, of Root Functions, of Rational Functions, and of Trigonometric Functions	
	– 2 × 20 –	40 points
	Total –	100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
(3) For draft paper, use pages in the answer booklet. If you use any other draft paper your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשע"ח, 2018, **מועד ב**
מספר השאלון: 035581
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה
5 יחידות לימוד - שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.

פרק ראשון —	אלגברה והסתברות	
	— 2×20	40 נקודות
פרק שני —	גאומטריה וטריגונומטריה במישור	
	— 1×20	20 נקודות
פרק שלישי —	חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות	
	— 2×20	40 נקודות
סה"כ —		100 נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון,
גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים,
בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון
או לפסילת הבחינה.
(3) לטיוטה יש להשתמש במחברת הבחינה.
שימוש בטיוטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The distance from Renana's home to her school is 500 meters.
Renana left home to go to school and walked at a constant speed.
3 minutes after she left the house, her father ran out to catch up with her and bring her the sandwich she had forgotten. He ran at a constant speed of 2.5 meters per second.

When her father caught up with Renana, they stood and chatted for 2 minutes, he gave her the sandwich, and then each of them continued on their way: Renana to school and her father back home. Renana continued walking at the same speed as before, and her father walked at a speed of 1.5 meters per second.

Renana's father arrived home at exactly the same time as Renana arrived at school.

א. Calculate Renana's walking speed.

ב. How much time passed from the moment Renana left home until she arrived at school?

2. The sequence a_n is defined for every natural n by the recursive rule:

$a_{n+1} = -\frac{c^{n-2}}{a_n}$, $a_1 = -\frac{1}{c}$. Given: $c > 0$.

א. Prove that the terms of the sequence a_n located in the odd positions form a geometric sequence, and that the terms of the sequence a_n located in the even positions also form a geometric sequence.

ב. **(1)** List the first 7 terms of the sequence a_n. Express your answer in terms of c if necessary.

(2) Express the sum of the first 7 terms in the sequence a_n in terms of c.

(3) Prove that for any natural n, the sum of the first $2n-1$ terms in the sequence a_n is independent of n.

ג. The sequence b_n is defined as: $b_n = -\frac{2}{a_n \cdot a_{n+1}}$.

(1) Show that b_n is a geometric sequence.

(2) What is the range of values of c for which b_n is a decreasing sequence?

(3) Given: The infinite sequence b_n is a decreasing sequence.
Express the sum of this sequence in terms of c.

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3. A multiple choice test contains 5 questions.

For each question 4 answers are given, but only one of them is correct.

Students are requested to mark one of the 4 answers given.

A student who marks a correct answer receives 20 points for that question.

A student who marks an incorrect answer does not receive any points for that question.

To pass the test, a student must score a total of at least 60 points.

א. Shahar was certain of the correct answer to 2 of the questions and marked them.

For the rest of the questions he randomly marked one answer for each question.

(1) What is the probability that Shahar will score exactly 60 points on this test?

(2) What is the probability that Shahar will pass the test?

ב. Daniel was certain of the correct answer to 2 of the questions and marked them.

For each of the remaining three questions, he was certain that one of the 4 given answers was incorrect, so in each of these questions he randomly marked one of the other answers.

What is the probability that Daniel will score exactly 60 points on this test?

ג. Hadas was certain of the correct answer to 3 of the questions and marked them.

For each of the remaining two questions, she was certain that k out of the 4 given answers were incorrect, so in each of these questions she randomly marked one of the other answers.

We know that the probability of Hadas scoring exactly 60 points on this test is equal to the probability of her scoring 100 points on the test.

Find k. Explain.

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Part Two — Planar Geometry and Planar Trigonometry (20 points)

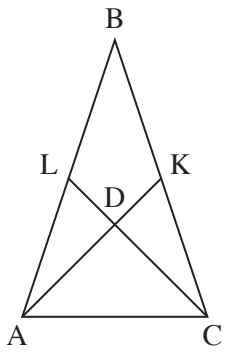
Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. ABC is an isosceles triangle (AB = BC).

AK and CL are medians of the triangle and they intersect at Point D.

Given: $AK \perp CL$.



א. Prove: BD = AC.

ב. Calculate the ratio $\frac{S_{BLDK}}{S_{\Delta ABC}}$.

ג. M is the center of the circle that circumscribes the quadrilateral ALKC.

(1) Prove: $\sphericalangle AML = 90^o$.

(2) Find the ratio $\frac{AM}{AD}$.

You may leave a square root sign in your answer.

5. ABC is an isosceles triangle (AB = AC).

BD is a an angle bisector in Triangle ABC.

The extension of Segment BD intersects the circle that circumscribes Triangle ABC at Point E.

The size of Angle ABC is 2β.

א. Express in terms of β the ratio $\frac{S_{\Delta ABC}}{S_{\Delta ADE}}$, which is the ratio between the area of Triangle ABC and the area of Triangle ADE.

There is no need to simplify the expression you obtained.

Given: BE is equal in length to the radius of the circle that circumscribes Triangle ABC.

ב. Calculate the ratio $\frac{S_{\Delta ABC}}{S_{\Delta ADE}}$.

Let a denote the length of Side AB.

ג. Express in terms of a the radius of the circle inscribed in Triangle ABC.

Your answers should be correct to two decimal places.

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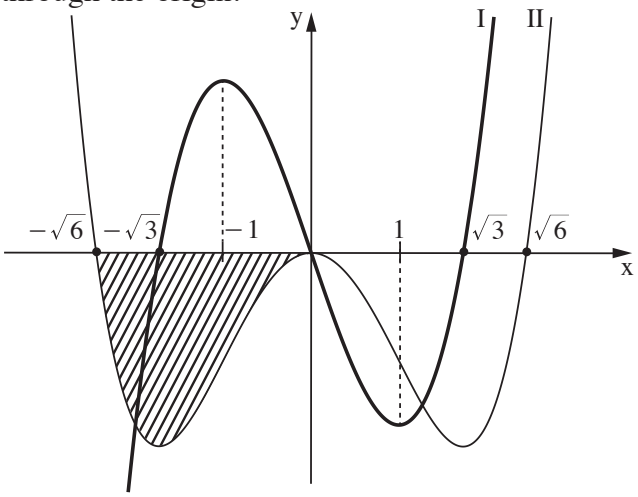
Part Three—Differential and Integral Calculus of Polynomials, of Root Functions, of Rational Functions, and of Trigonometric Functions

(40 points)

Answer two of the questions 6-8 (each question – 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Shown below are the graphs of the functions $f'(x)$ and $f''(x)$ (the function of the first derivative and the function of the second derivative of the function $f(x)$) over the interval $-2.5 \le x \le 2.5$. Both graphs pass through the origin.



א. Match the Graphs I and II with the functions $f'(x)$ and $f''(x)$. Explain.

ב. **(1)** How many internal extrema [נקודות קיצון פנימיות] does the function $f(x)$ have in the interval shown in the graph? Explain your answer.

(2) How many inflection points [נקודות פיתול] does the function $f(x)$ have in the interval shown in the graph? Explain your answer.

ג. For which value of x in the interval $-\sqrt{3} \le x \le \sqrt{3}$ is the slope of the tangent to the graph of the derivative function $f'(x)$ smallest [מינימלי]?

Given: $f(x)$ is an odd function.

ד. Sketch a graph of the function $f(x)$.

Given: The value of function $f(x)$ at its maximum point is t.

ה. Express in terms of t the area bounded by Graph II and by the negative side of the x-axis (the shaded area in the figure).

ו. Given: There are real a, b, and c such that: $f(x) = ax^5 + bx^3 + c$.
Find c and the ratio $\frac{a}{b}$.

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7. Given: The function $f(x) = \sin\left(\frac{\pi}{x}\right)$.

א. What is the domain [תחום ההגדרה] of the function f(x) ?

Answer Items ב-ה for the interval $x \geq \frac{2}{7}$.

ב. Find the coordinates of the points at which the graph of the function f(x) intersects the x-axis.

ג. Find the coordinates of the extrema [נקודות הקיצון] of function f(x), and determine what type of extrema they are.

ד. Function f(x) has a horizontal asymptote. Find the equation of the horizontal asymptote of the function f(x).

ה. Sketch a graph of the function f(x).

Answer Item ו for the interval $x > 0$.

ו. Consider the points at which the graph of the function f(x) intersects the x-axis.
Read the following statements (i-iii). One of them is correct. Which one is correct? Explain.

i) The closer we get to $x = 0$, the smaller the distance between two adjacent intersection points.

ii) The distance between every two adjacent intersection points is constant.

iii) The closer we get to $x = 0$, the greater the distance between two adjacent intersection points.

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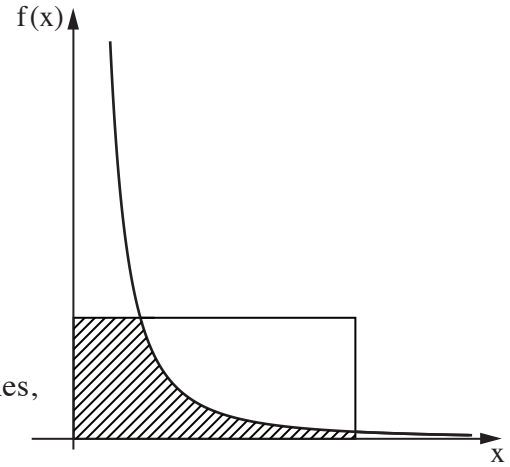
8. The figure at right shows the graph of the function $f(x)=\frac{1}{x^2}$ over the interval $x>0$, and a rectangle of which two sides lie on the axes. The rectangle is positioned in the first quadrant.



Given: The area of the rectangle is 4.

Let a be the length of the rectangle side that lies on the x-axis. Given: $a \geq \frac{1}{4}$.

א. Express in terms of a the area bounded by the axes, the sides of the rectangle, and the graph of the function f(x) (the shaded area in the figure).

ב. What is the value of a for which the area you found in Item א is maximal (greatest)?

Good Luck!