

Mathematics

4 Points — 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes

ב. Exam structure and breakdown of points:

This exam has two parts.

Part One — Sequences, Spatial Trigonometry

$$- 1 \times 33\frac{1}{3} - 33\frac{1}{3} \text{ points}$$

Part Two — Growth and Decay, Differential and Integral

Calculus of Trigonometric Functions,

Exponential and Logarithmic Functions,

and Power Functions

$$- 2 \times 33\frac{1}{3} - 66\frac{2}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
- (3) For draft paper, use the answer booklet. If you use other draft paper, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון — סדרות, טריגונומטריה במרחב

$$- 1 \times 33\frac{1}{3} - 33\frac{1}{3} \text{ נקודות}$$

פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של

פונקציות טריגונומטריות,

פונקציות מעריכיות ולוגריתמיות

ופונקציות חזקה

$$- 2 \times 33\frac{1}{3} - 66\frac{2}{3} \text{ נקודות}$$

$$\text{סה"כ} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או

לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Sequences, Spatial Trigonometry ($33\frac{1}{3}$ points)

Answer one of the questions 1-2.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

Sequences

1. Given: two infinite geometric sequences, a_n and b_n .

The common ratio of Sequence a_n is q , and the common ratio of Sequence b_n is $3q$.

Given: $a_1 = b_1$.

Let S be the sum of the terms of Sequence a_n , and

let T be the sum of the terms of Sequence b_n (S and T are real numbers).

Given: $\frac{S}{T} = \frac{6}{7}$.

א. Calculate q .

Given: $a_4 = 5$.

ב. Calculate b_4 .

Spatial Trigonometry

2. $ABCD A' B' C' D'$ is a cube. The length of each of its edges is a (see figure).

א. Explain why Triangle $A'BD$ is an equilateral triangle.

$A'M$ is a height in Triangle $A'BD$.

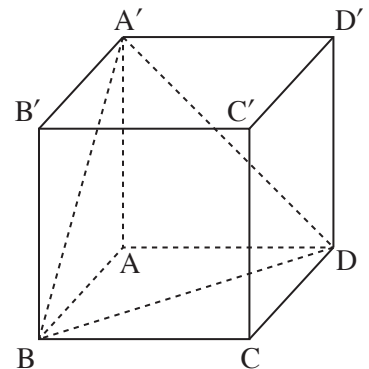
ב. Calculate the size of the angle between $A'M$ and Face $ABCD$.

Given: The area of Triangle $A'BD$ is $8\sqrt{3}$.

ג. (1) Calculate a .

(2) $AA'BD$ is a pyramid. Calculate its surface area.

Your answer should be correct to two decimal places.



Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions
($66\frac{2}{3}$ points)

Answer two of the questions 3-5 (each question — $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

3. Given: the function $f(x) = 2 \cdot \sin x + \cos(2x)$, which is defined over the interval $0 \leq x \leq \frac{\pi}{2}$.
Answer Items א and ב for the given interval.
- א. Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and determine what type of extrema they are.
- ב. Sketch a graph of the function $f(x)$.

The line $y = k$ is tangent to the graph of the function $f(x)$ in the given interval at the function's maximum point.

- א. (1) Find k .
- (2) Calculate the area bounded by the graph of the function $f(x)$, the y -axis, the line $y = k$, and the line $x = \frac{\pi}{2}$.
4. Given: the function $f(x) = \frac{a - e^x}{e^{2x}}$. a is a parameter, $a > 0$.
- א. (1) What is the domain [תחום ההגדרה] of the function $f(x)$?
- (2) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes (if there are such points). Express in terms of a , if necessary.

Given: The graph of the function $f(x)$ passes through the origin.

- ב. Find a .

Substitute the value of a that you found, and answer Items ג and ד.

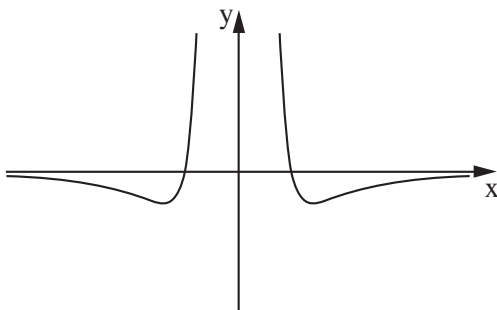
- ג. (1) Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$, and determine what type of extremum it is.
- (2) Sketch a graph of the function $f(x)$.

The function $g(x)$ satisfies: $g'(x) = f(x)$.

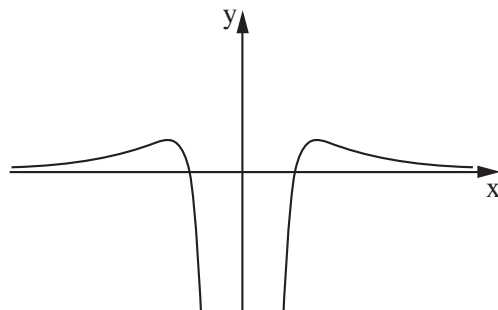
- ד. Find the x -coordinate of the extremum of the function $g(x)$, and determine what type of extremum it is.

5. Given: the function $f(x) = \frac{\ln(x^2)}{x^2}$.

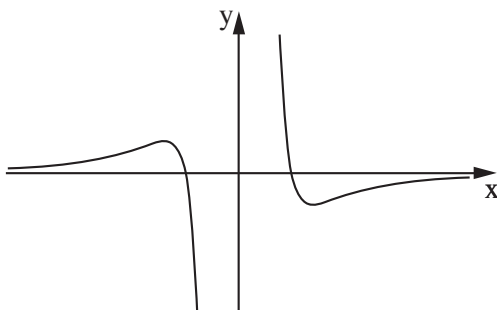
- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
 (2) Find the equation of the vertical asymptote of the function $f(x)$.
 (3) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes (if there are such points).
 (4) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and determine what type of extrema they are.
 (5) Sketch a graph of the function $f(x)$.
 (6) Find the intervals on which the function $f(x)$ is positive and negative.
- ב. Shown below are four graphs (I - IV). Which of them is the graph of the derivative function, $f'(x)$? Explain.



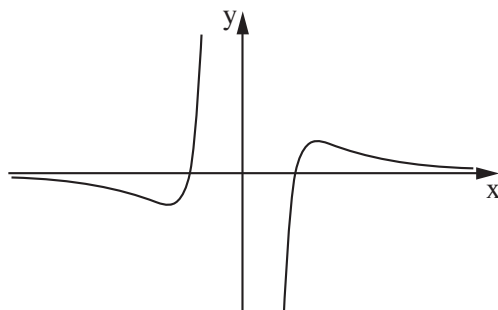
II



I



IV



III

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך