

Mathematics

4 Points — 1st Exam

Instructions for examinees

- א. Duration of the exam: Three and a half hours
- ב. Exam structure and breakdown of points:
This exam has three parts.
- Part One — Algebra, Analytical Geometry, Probability
— 2×20 — 40 points
- Part Two — Planar Geometry and Planar Trigonometry
— 1×20 — 20 points
- Part Three — Differential and Integral Calculus of
Polynomials, Rational Functions, and
Root Functions
— 2×20 — 40 points
Total — 100 points
- ג. Material that may be used during the exam:
- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.
- ד. Special instructions:
- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
- (3) For draft paper, use pages in the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

- א. משך הבחינה: שלוש שעות וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון — אלגברה, גאומטריה אנליטית, הסתברות
— 2×20 — 40 נקודות
פרק שני — גאומטריה וטריגונומטריה במישור
— 1×20 — 20 נקודות
פרק שלישי — חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש
— 2×20 — 40 נקודות
סה"כ — 100 נקודות
- ג. חומר עזר מותר בשימוש:
- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי-לועזי/לועזי-עברי.
- ד. הוראות מיוחדות:
- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון,
גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
- (3) לטייטה יש להשתמש במחברת הבחינה.
שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

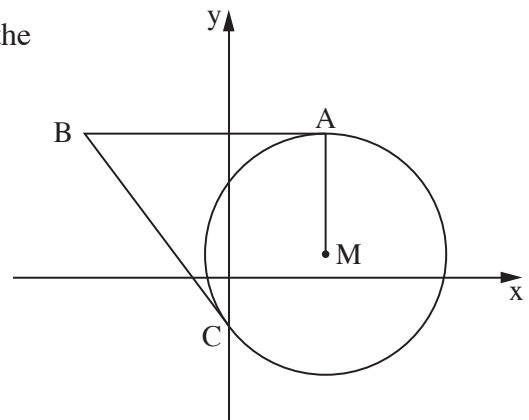
Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. In an art class, a student received an iron wire of length $52a$ cm and prepared from it two picture frames: one square frame and one rectangular frame.
One side of the rectangle is equal in length to the side of the square, and the other side of the rectangle is $\frac{4}{3}$ larger than the side of the square.
The wire was exactly long enough to prepare the two frames.
 - א. Express the lengths of the sides of the rectangle in terms of a .
 - ב. The student prepared two more frames from an additional iron wire (of different length): a rectangular frame identical to the first rectangular frame, and a square frame whose side was 65% longer than the side of the first square.
By how many percent is the second wire longer than the first wire?
 - ג. The length of the diagonal of the rectangle is 45 cm.
Calculate the lengths of the sides of the rectangle.

2. A circle, whose center is at Point M (4, 1), intersects the y-axis at Point C, as shown in the figure.
From Point B, which is located in the second quadrant, two lines were drawn that are tangent to the circle at Points A and C.
The equation of Line AB is $y = 6$.
 - א. What is the equation of the circle?
 - ב. Find the equation of Line BC.
 - ג. Calculate the area of Quadrilateral ABCM.
 - ד. Calculate the radius of the circle that circumscribes Triangle BCM.
Your answer should be correct to two decimal places.



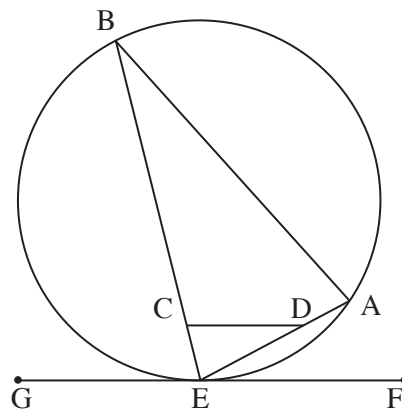
3. A certain school has students who live in the city and students who live outside the city. The number of girls at the school is 1.25 times greater than the number of boys at the school. 75% of the boys live in the city, and 40% of the girls live outside the city. A student (boy or girl) was chosen at random.
- א. What is the probability that the chosen student was a boy who lives in the city?
 - ב. We know that the chosen student (boy or girl) lives in the city. What is the probability that a girl was chosen?
 - ג. The school has 900 students. How many students live in the city?
 - ד. Each day, a student is chosen at random for cleanup duty (the same student can be chosen on consecutive days). What is the probability that on 3 consecutive days, at least 2 students who live outside the city were chosen?

Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Triangle AEB is inscribed in a circle. Segment GF is tangent to the circle at Point E. Points C and D are located on Sides BE and AE, respectively, such that Segment CD is parallel to the tangent.
- א. Prove: $\angle ABE = \angle CDE$.
 - ב. Prove: $\triangle CDE \sim \triangle ABE$.
 - ג. Prove that it is possible to inscribe Quadrilateral ABCD in a circle.
 - ד. Given: $CD = 4$ cm, $BE = 12$ cm, $ED = \frac{1}{3}AB$. Calculate the length of Segment ED.



5. ABD is a right triangle ($\angle ABD = 90^\circ$).

Let $BD = a$. Given: $AB = 3a$.

א. Calculate the size of Angle ADB.

C is a point outside the triangle.

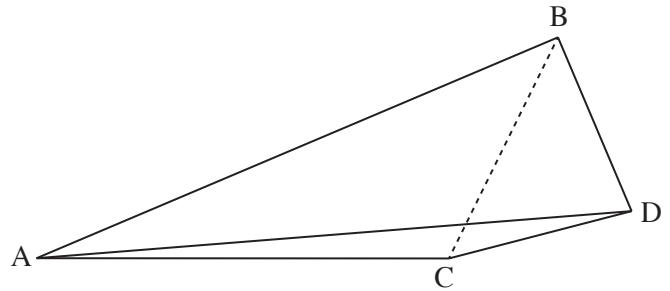
Given: $\angle ADC = 10^\circ$, $CD = BD$.

ב. Express the length of Segment BC in terms of a .

ג. Express the length of Segment AC in terms of a .

ד. Given: The area of Triangle BDC is 30 cm^2 .

Calculate the area of Quadrilateral ABDC.



Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = x^2(x - 4)^2$, which is defined for every x .

Answer Items ג-א. Remove parentheses if necessary.

א. (1) Find the coordinates of the points where the graph of function $f(x)$ intersects the axes.

(2) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and determine what type of extrema they are.

(3) Sketch a graph of the function $f(x)$.

(4) Find the intervals on which the function $f(x)$ is positive and negative (if there are such intervals).

ב. Calculate the area bounded by the graph of the function $f(x)$ and the x -axis.

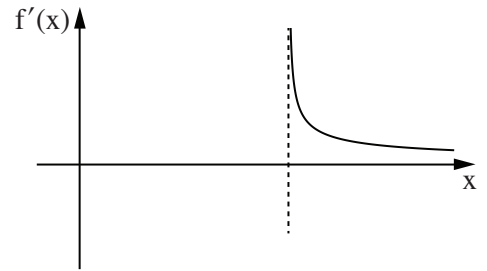
ג. Sketch a graph of the derivative function, $f'(x)$.

7. Given: the function $f(x) = \sqrt{2x - 13}$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the coordinates of the points where the graph of function $f(x)$ intersects the axes (if there are such points).
- (3) Show that the function $f(x)$ increases over its entire domain.
- (4) Sketch a graph of the function $f(x)$.

Shown at right is the graph of the derivative function, $f'(x)$.

- א. (1) What is the domain of the derivative function, $f'(x)$?
- (2) What is the equation of the vertical asymptote of the derivative function, $f'(x)$?



The graphs of the functions $f(x)$ and $f'(x)$ intersect at Point A.

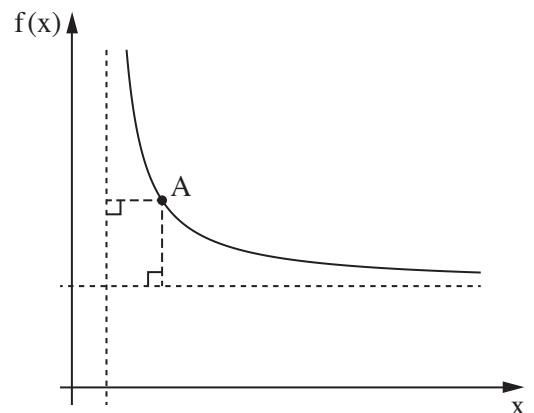
- א. Calculate the coordinates of Point A.

A perpendicular was dropped from Point A to the x-axis.

- א. Calculate the area bounded by the graph of the derivative function $f'(x)$, the perpendicular, the x-axis, and the line $x = 11$.

8. Shown below is the graph of the function $f(x) = \frac{4}{x-1} + 3$ in the first quadrant. From Point A, which is located on the graph of the function $f(x)$ in the first quadrant, perpendiculars were drawn to the asymptotes of the function $f(x)$, so that a rectangle was formed.

- א. Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes.
- א. Find the coordinates of Point A for which the perimeter of the rectangle is minimal (smallest).
- א. Calculate the area of the rectangle whose perimeter is minimal.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך