

Mathematics

3 Points — 3rd Exam

Instructions for examinees

א. Duration of the exam: Two hours

ב. Exam structure and breakdown of points:

This exam has six questions in the topic areas:

Algebra, Differential and Integral Calculus.

You must answer four questions — $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of your exam.
- (3) For draft paper, use the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שעותיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות — $4 \times 25 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
- (3) לטייטה יש להשתמש במחברת הבחינה. שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-6 (each question – 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. A book store announced a sale:

If you buy two books, you get a 50% discount on the cheaper of the two books.

א. Orit bought two books at the sale. Their prices before the sale were 108 shekels and 72 shekels.

(1) Calculate how many shekels Orit paid for the two books.

(2) Calculate the total discount, in percent, that Orit received for the two books together.

ב. Ze'ev also bought two books at the same sale and paid a total of 165 shekels.

Before the sale, the price of the more expensive book was 39 shekels higher than the price of the cheaper book.

(1) Calculate the pre-sale price of each of the two books that Ze'ev bought.

(2) Calculate the total discount, in percent, that Ze'ev received for the two books together.

Your answer should be correct to two decimal places.

2. AEB is a right triangle ($\angle AEB = 90^\circ$).

Vertex A is located on the y-axis (see figure).

The equation of Side AE is $y = -\frac{1}{2}x + 5$.

א. Find the coordinates of Vertex A.

Given: The extension of Side BE passes through the origin, O.

ב. Find the equation of Line OB.

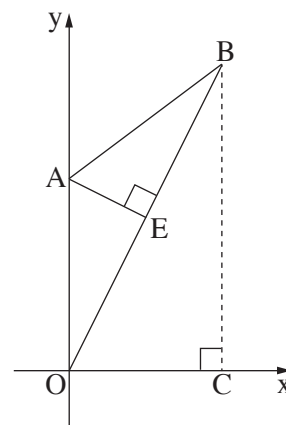
ג. Find the coordinates of Point E.

Given: The y-coordinate of Vertex B is 8.

ד. Show that Triangle OAB is an isosceles triangle.

A line was drawn from Point B, perpendicular to the x-axis and intersecting it at Point C.

ה. Calculate the perimeter of Quadrilateral ABCO.



3. Given: a circle whose center is at the point M(3, 5) and whose radius is R.

A line was drawn tangent to the circle at Point A(1, 8),

as shown in the figure.

א. (1) Calculate the radius R of the circle.

(2) Write the equation of the circle.

ב. (1) Find the slope of Line AM.

(2) Find the equation of the tangent.

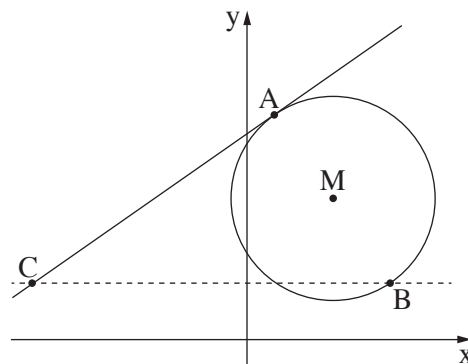
Given: AB is a diameter of the circle.

ג. Find the coordinates of Point B.

A line parallel to the x-axis was drawn through Point B (the dashed line in the figure).

The line intersects the tangent at Point C.

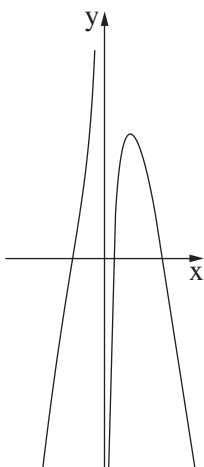
ד. Calculate the area of Triangle ABC.



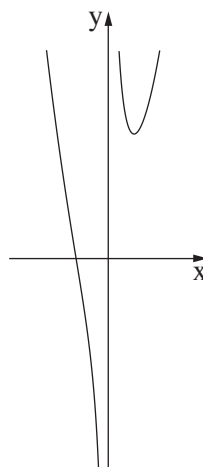
Differential and Integral Calculus

4. Given: The function $f(x) = 0.5x^2 + \frac{8}{x}$.

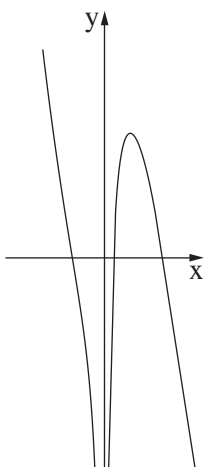
- א. What is the domain [תחום ההגדרה] of function $f(x)$?
- ב. Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what kind of extremum it is.
- ג. Is function $f(x)$ increasing or decreasing at the point where $x = -1$? Explain.
- ד. Four graphs (I-IV) are shown below. Which of them is the graph of function $f(x)$? Explain.



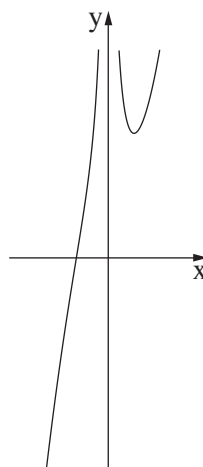
II



I



IV



III

Continued on page 5 ►

5. The figure at the right shows the graphs of the functions

$$f(x) = x^2 - 4x + 6, \quad g(x) = -2x + 14.$$

The two graphs intersect at Point B(4, 6).

Point A is the minimum point of function $f(x)$.

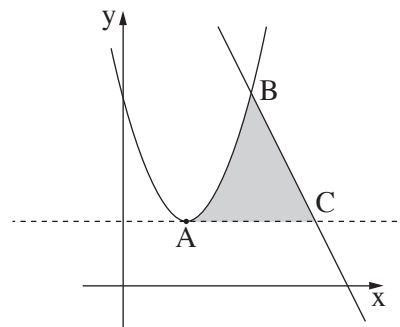
א. Find the coordinates of Point A.

The line $y = 2$ (the dashed line in the figure) is tangent to the graph of function $f(x)$ at Point A.

The tangent line intersects the graph of function $g(x)$ at Point C (see figure).

ב. Find the coordinates of Point C.

ג. Find the area of the grey region in the figure, which is bounded by the graphs of functions $f(x)$ and $g(x)$ and by the line $y = 2$.



6. The figure at the right shows two graphs, whose equations are:

I. $y = x$

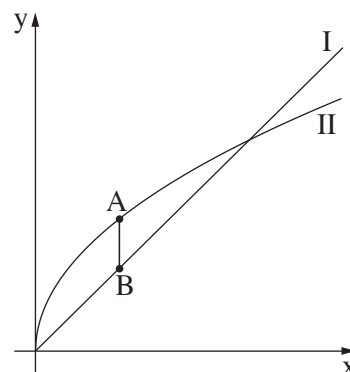
II. $y = \sqrt{x}$

Point A is located on Graph II and Point B is located on Graph I, such that the segment AB is parallel to the y-axis.

Points A and B are located between the intersection points of the graphs, as shown in the figure.

א. Find the value of the x-coordinate of Point A for which the length of Segment AB will be maximal.

ב. Calculate the maximum length of Segment AB.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך