

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors,

Spatial Trigonometry, Complex Numbers

$$2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ points}$$

Part Two: Growth and Decay, Power Functions,

Exponential and Logarithmic Functions

$$1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ points}$$

$$\text{Total} \quad \text{---} \quad 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.

Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or

lead to the disqualification of your exam.

(3) For draft paper, use the pages in the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

$$2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ נקודות}$$

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

$$1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ נקודות}$$

$$\text{סה"כ} \quad \text{---} \quad 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה מסודרת.

חסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטיוטה יש להשתמש במחברת הבחינה.

שימוש בטיוטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Given: Points $A(-3a, 0)$ and $B(3, 0)$. $a > 0$ is a parameter.

- א. Express in terms of a the locus [המקום הגאומטרי] of all the points P that satisfy $\frac{PA}{PB} = 1$.
- ב. Show that the locus of all the points Q that satisfy $\frac{QA}{QB} = 2$ is a circle, and express in terms of a the coordinates of the center of this circle and its radius.
- ג. Consider the set of all the circles that are tangent to the locus that you found in Item א and pass through the center of the circle that you found in Item ב.

Given: The centers of these circles form a locus that passes through the origin of the axes.

- (1) Identify this locus.
- (2) Find a , and write the equation of this locus.

2. The figure below shows the cuboid $ABCD A' B' C' D'$.

Edges DA , DC , and DD' lie on the x , y , and z axes, respectively, as shown in the figure.

Given: $DA = 4$, $AA' = 3$.

Let $AB = a$. $a > 0$ is a parameter.

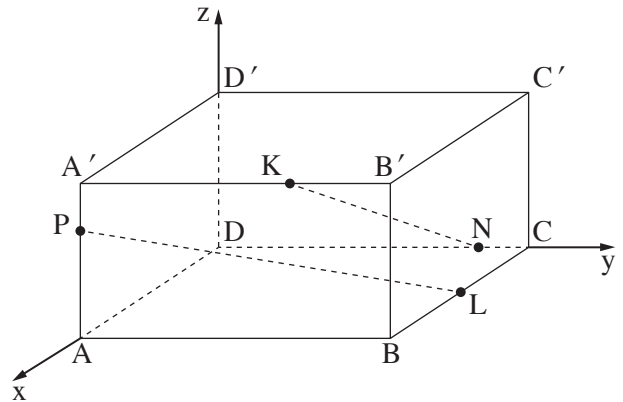
Points P , N , L , and K are located on the cuboid edges AA' , DC , BC , and $A'B'$, respectively, such that the following are true:

$$AP = 2PA',$$

$$N(0, 5, 0),$$

L is the midpoint of Edge BC ,

$$\overrightarrow{A'K} = \frac{4}{5}\overrightarrow{DN}.$$



א. Find the equation of Plane PNK .

א. (1) Find parametric representations of Line NK and of Line PL .

If necessary, express them in terms of a .

(2) Explain why Lines NK and PL are skew lines [ישרים מצטלבים].

א. (1) For what value of a is $\angle PC'C = 82.1^\circ$?

Your answer should be correct to two decimal places.

(2) Is there a value of a for which $\angle PC'C = 90^\circ$? Explain.

3. z_1 and z_2 are two complex numbers that satisfy: $|z_1| = |z_2| = r$, $\arg z_1 + \arg z_2 = 90^\circ$.

א. Prove that the product of $z_1 \cdot z_2$ is a pure imaginary number, and express it in terms of r .

Points A , B , and C in the Gauss plane represent the complex numbers z_1 , z_2 , and z_3 , respectively.

Given: Points A , B , and C are not located on one line, and Point C is located on the line $y = x$.

א. Explain why Triangle ABC is an isosceles triangle.

Point D in the Gauss plane corresponds to the complex number $z_3 \cdot (z_1 \cdot z_2)^2$.

Given: $z_1 + z_2 = 7 + 7i$

$$z_1 - z_2 = 1 - i$$

$$(z_3)^2 = 2i$$

א. (1) Find the coordinates of Points C and D (find both possibilities).

(2) Calculate the area of the quadrilateral $BDAC$ for the point C that is located in the first quadrant.

Continued on page 4 ►

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: the family of functions $f(x) = e^{2mx} - e^{mx}$. $m > 0$ is a parameter.

Answer Item א. If necessary, express in terms of m .

- א. (1) What is the domain [תחום ההגדרה] of the function $f(x)$?
- (2) Find the coordinates of the points of intersection of the graph of the function $f(x)$ with the axes (if there are any).
- (3) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes (if there are any).
- (4) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$ (if there are any), and determine what type of extrema they are.
- ב. On a single set of axes, sketch the graphs of the functions $f(x)$ for $m = 1$ and for $m = 2$ (it is known that the two graphs intersect each other at exactly two points). Show which graph corresponds to each of the given values of m .

The line $y = k$ is tangent to the graph of the function $f(x)$.

- ג. (1) Express in terms of m the area of the region bounded by the graph of the function $f(x)$, the y -axis, and the line $y = k$.

For each m , we denote the area described in Sub-item ג(1) by S_m .

(S_1 is the area for $m = 1$).

- (2) Show that for every m , the relation $S_m = \frac{S_1}{m}$ is satisfied.

5. Given: $f(x)$ is a function that is continuous and differentiable [פונקציה רציפה וגזירה] for all x .

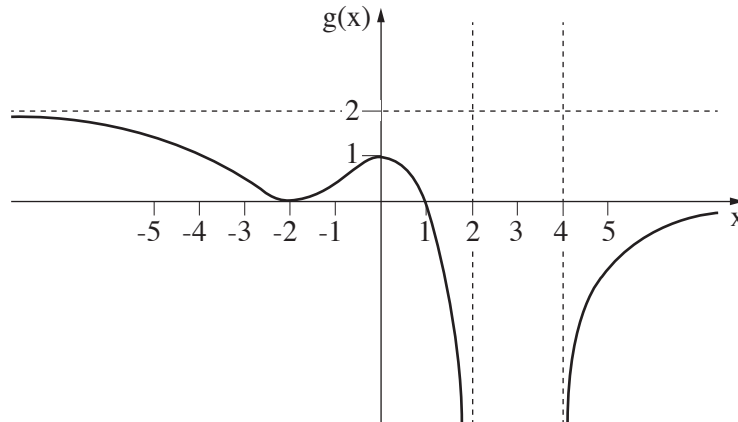
Let $g(x) = \ln(f(x))$.

The function $g(x)$ is defined for all $x > 4$ and for all $x < 2$, and only there.

In the interval $2 \leq x \leq 4$, $f'(x) = 0$ only for $x = 3$.

The figure below shows the graph of the function $g(x)$.

The function $g(x)$ equals zero only at the points where $x = 1$ and $x = -2$.



- א. Find $f(-2)$, $f(0)$, and $f(1)$.
- ב. What are the intervals on which the function $f(x)$ is positive and on which it is negative? Explain.
- ג. What are the coordinates of the points of intersection of the graph of the function $f(x)$ with each of the axes? Explain.
- ד. The function $f(x)$ has two horizontal asymptotes. Write their equations.
- ה. What are the intervals on which the function $f(x)$ is increasing and on which it is decreasing? Explain.
- ו. Sketch the graph of the function $f(x)$.
- ז. Explain why: $\int_{-2}^1 f(x) dx > 3$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך