

Mathematics

4 Points – 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes

ב. Exam structure and breakdown of points:

This exam has two parts.

Part One — Sequences, Spatial Trigonometry

$$- 1 \times 33\frac{1}{3} - 33\frac{1}{3} \text{ points}$$

Part Two — Growth and Decay, Differential and Integral

Calculus of Trigonometric Functions,

Exponential and Logarithmic Functions, and

Power Functions

$$- 2 \times 33\frac{1}{3} - 66\frac{2}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
- (3) For draft paper, use the answer booklet.
If you use other draft paper, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון — סדרות, טריגונומטריה במרחב

$$- 33\frac{1}{3} \times 1 - 33\frac{1}{3} \text{ נקודות}$$

פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של

פונקציות טריגונומטריות,

פונקציות מעריכיות ולוגריתמיות

ופונקציות חזקה

$$- 33\frac{1}{3} \times 2 - 66\frac{2}{3} \text{ נקודות}$$

$$\text{סה"כ} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות

במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול

לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Sequences, Spatial Trigonometry ($33\frac{1}{3}$ points)

Answer one of the questions 1-2.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

Sequences

1. The sequence a_n is defined as follows: $a_{n+1} = a_n + 3$, $a_1 = 0$.

The sequence b_n is defined by the relation: $b_n = a_n + a_{n+1}$.

א. (1) Prove: $b_n = 2a_n + 3$.

(2) Prove that the sequence b_n is an arithmetic sequence, find its common difference, and find b_1 .

Given: $b_1 + b_m = 120$.

א. (1) Calculate m .

(2) Calculate the sum: $b_{m+1} + b_{m+2} + \dots + b_{2m}$

(the sum of the terms in the sequence b_n , from b_{m+1} through b_{2m} , inclusive).

Spatial Trigonometry

2. ABCDE is a right pyramid, whose base is a square, as shown in the figure.

Given: EM is a height of the pyramid. Its length is equal to that of the diagonal of the base.

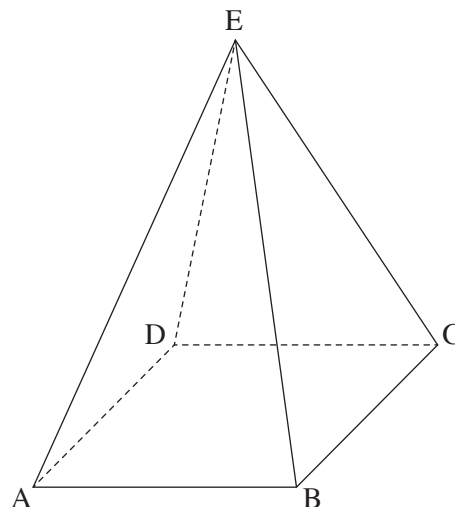
Let $AB = a$.

א. Calculate the size of the angle between a side edge of the pyramid and the base of the pyramid.

EK is a height of a side face of the pyramid.

א. Calculate the size of the angle between EK and the base of the pyramid.

א. Given: The lateral surface area [שטח מעטפת] of the pyramid is 36.75 cm^2 . Calculate a .



**Part Two — Growth and Decay, Differential and Integral Calculus of
Trigonometric Functions, Exponential and Logarithmic Functions,
and Power Functions** ($66\frac{2}{3}$ points)

Answer two of the questions 3-5 (each question — $33\frac{1}{3}$ points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

3. The function $f'(x) = 2 \sin 2x$ is the derivative function of the function $f(x)$.

Answer Items א-ג for the interval $0 \leq x \leq \pi$.

- א. Find the x -coordinates of the extrema [נקודות הקיצון] of the function $f(x)$ in the given interval, and determine what type of extrema they are.

The graph of the function $f(x)$ passes through the point $(0, -2)$.

- ב. Find the function $f(x)$.
ג. Find the coordinates of the point at which the graph of the function $f(x)$ intersects the x -axis.
ד. Sketch the graph of the function $f(x)$.
ה. Calculate the area of the region bounded by the graph of the function $f(x)$, the line $x = \pi$, the y -axis, and the x -axis.

4. Given: the function $f(x) = ae^x - 9e^{-x}$. a is a parameter.

- א. Find the domain [תחום ההגדרה] of the function $f(x)$.

The slope of the tangent to the graph of the function $f(x)$ at the point at which $x = \ln 3$ is 6.

- ב. Find a . Show your calculations in detail.

Substitute $a = 1$, and answer Items ג and ד.

- ג. (1) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes.
(2) Find the intervals on which the function $f(x)$ is increasing and decreasing (if there are any such intervals).
(3) Sketch the graph of the function $f(x)$.
ד. Calculate the area of the region bounded by the the graph of the function $f(x)$ and the axes.

5. Given: the function $f(x) = \frac{2x}{\ln(x) - 2}$.

- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
- ב. (1) Does the graph of the function $f(x)$ intersect the axes? If so, find the points of intersection. If not, explain.
- (2) The function $f(x)$ has one vertical asymptote. Find its equation.
- (3) Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$, and determine what type of extremum it is.
- (4) What are the intervals on which the function $f(x)$ is increasing and decreasing?
- (5) Calculate $f(0.1)$ and sketch the graph of the function $f(x)$.

The function $g(x)$ satisfies $g'(x) = f(x)$.

- ג. What is the interval on which the function $g(x)$ is increasing?

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך