

**The State of Israel
Ministry of Education**

Exam Type: *Bagrut*

Exam Date: Summer 2018

Exam Number: 035481

Appendix: Formula sheets for 4 points

English Translation (3)

מדינת ישראל

משרד החינוך

סוג הבחינה: בגרות

מועד הבחינה: קיץ תשע"ח, 2018

מספר השאלון: 035481

נספח: דפי נוסחאות ל-4 יחידות לימוד

תרגום לאנגלית (3)

Mathematics

4 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

Probability — 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus of

Polynomials, of Rational Functions, and of

Root Functions

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

(3) For draft paper, use pages in the answer booklet.

Use of other draft paper may lead to the disqualification of your exam.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית,

הסתברות — 2×20 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

— 1×20 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

של פולינומים, של פונקציות רציונליות

ושל פונקציות שורש

— 2×20 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.

רשום במחברת את שלבי הפתרון,

גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry, Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The distance between City א and City ב is 126 km.

At 8:00 AM, a car left City א, heading for City ב.

At 8:30 AM, a cyclist left City ב, heading for City א.

The car and the cyclist met at 9:30 AM, and both continued on their ways.

15 minutes after they met, the car arrived at City ב.

The car and the cyclist did not change their speeds during their trips.

א. Find the speed of the car and the speed of the cyclist.

The next day, the car and the cyclist began traveling towards each other at the same time.

The car left City ב, heading for City א, whereas the cyclist left City א, heading for City ב.

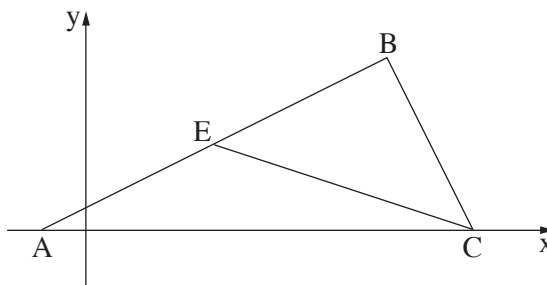
The car traveled at a constant speed that was a kilometers per hour greater than the speed at which it had traveled the previous day, whereas the cyclist rode at a constant speed that was a kilometers per hour less than the speed at which he had ridden the previous day.

The car and the cyclist met after t hours.

- ב. Find t .

2. CE is a median in Triangle ABC.

Given: $A(-1, 0)$, $B(7, 4)$,
Vertex C is located on the x-axis
(see figure).



א. Find the coordinates of Point E.

Given: $EB = BC$,
the x-coordinate of Vertex C is larger than the x-coordinate of Vertex B.

ב. Find the coordinates of Vertex C.

A perpendicular was drawn from Point B to the x-axis.

This perpendicular intersects Segment CE at Point K and the x-axis at Point F.

ג. (1) Find the coordinates of Point K and the length of Segment KF.

(2) Calculate the area of Triangle EKF.

3. A basket contains 2 apples and a certain number of peaches.

Tal removed two pieces of fruit from the basket at random, one after the other,
without returning either of them.

The probability that she removed two apples is $\frac{1}{36}$.

א. Find how many peaches were in the basket before Tal removed any fruit.

ב. What is the probability that the second piece of fruit that Tal removed was an apple?

ג. (1) Calculate the probability that Tal removed from the basket two pieces of fruit of the same kind.

(2) We know that Tal removed from the basket two pieces of fruit of the same kind.
What is the probability that she removed two peaches?

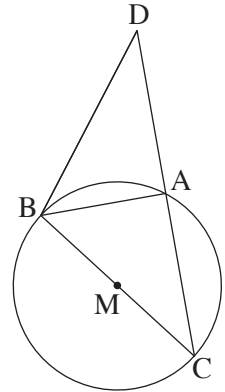
Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note: If you answer more than one question, only the first answer in your answer booklet will be graded.

4. The figure at the right shows a circle of radius R whose center is M .
 BC is a diameter of the circle. Point D is located outside the circle.
 Segment DC intersects the circle at Point A .

Given: $\angle ABD = \frac{1}{2} \angle AMC$.



- א. Prove that BA is an angle bisector in Triangle DBC .
- ב. Prove: $\triangle CBD \sim \triangle CMA$.
- ג. Prove that MA is a midsegment [קטע אמצעים] in Triangle DBC .
- ד. Given: Triangle ABM is an equilateral triangle.

Express the area of Triangle CBD in terms of the radius of the circle.

5. $ABCD$ is a parallelogram.

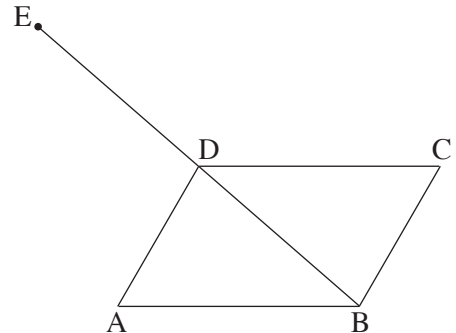
Given: $AB = 15$, $BC = 10$.

Let: $\angle DAB = \alpha$ ($\alpha < 90^\circ$).

- א. Express the area of Triangle BAD in terms of α .

Given: The area of the parallelogram is $75\sqrt{3}$.

- ב. Calculate the size of the angle α .
- ג. Calculate the length of Diagonal BD .



Point E is located on the extension of Diagonal BD , as shown in the figure,
 such that $ED = DB$.

- ד. (1) Find the size of Angle ABE .
- (2) Find the radius of the circle that circumscribes Triangle ABE .

Part Three — Differential and Integral Calculus of Polynomials, of Rational Functions, and of Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note: If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: the function $f(x) = \frac{1}{(x-3)^2} + 4$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes.
- (3) Find the intervals on which the function $f(x)$ is increasing and decreasing.
- (4) Sketch the graph of the function $f(x)$.
- ב. Calculate the area of the region bounded by the graph of the function $f(x)$, the x -axis, and the lines $x = 4$ and $x = 5$.

Given: the function $g(x) = f(x) - 4$.

- ג. What is the area of the region bounded by the graph of the function $g(x)$, the x -axis, and the lines $x = 4$ and $x = 5$? Explain.

7. Given: the function $f(x) = x^3 \cdot \sqrt{x+a}$. a is a parameter.

- א. Express the domain [תחום ההגדרה] of the function $f(x)$ in terms of a .

The point $(2, 24)$ is located on the graph of the function $f(x)$.

- ב. Find a .

Substitute $a = 7$ and answer Items ג and ד.

- ג. (1) Find the coordinates of the points of intersection between the graph of the function $f(x)$ and the axes.
- (2) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and determine what type of extrema they are.
- (3) Sketch the graph of the function $f(x)$.
- (4) What are the intervals on which the function $f(x)$ is positive and negative?

Given: the function $g(x) = f(x) + c$. c is a parameter.

- ד. What is the value of c for which the graph of the function $g(x)$ is tangent to the x -axis? Explain.

Continued on page 6 ►

8. The figure at the right shows the graph of the function $f(x) = x^3$ and the line $x = 2$.

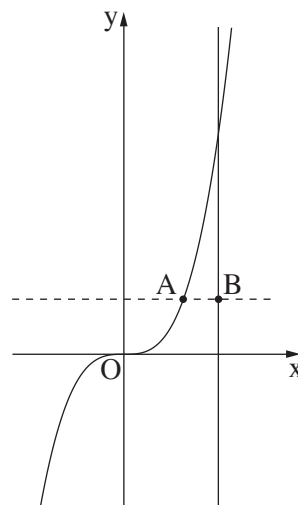
Point A is located on the graph of the function $f(x)$,

Given: $0 < x_A < 2$ (x_A is the x-coordinate of Point A).

A line parallel to the x-axis was drawn from Point A (the dashed line in the figure).

This line intersects the line $x = 2$ at Point B (see figure).

Point O is the origin.



- א. What are the coordinates of Point A for which the area of Triangle ABO is maximal (largest)? Explain.
- ב. Calculate the area of Triangle ABO for the Point A that you found in Item א.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך