

Mathematics

4 Points — 2nd Exam

Instructions for examinees

א. Duration of the exam: One hour and 45 minutes

ב. Exam structure and breakdown of points:

This exam has two parts.

Part One — Sequences, Spatial Trigonometry

$$- 1 \times 33\frac{1}{3} - 33\frac{1}{3} \text{ points}$$

Part Two — Growth and Decay, Differential and Integral

Calculus of Trigonometric Functions,
Exponential and Logarithmic Functions, and
Power Functions

$$- 2 \times 33\frac{1}{3} - 66\frac{2}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
- (3) For draft paper, use the answer booklet. If you use other draft paper, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון — סדרות, טריגונומטריה במרחב

$$- 1 \times 33\frac{1}{3} - 33\frac{1}{3} \times 1 \text{ נקודות}$$

פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי

ואינטגרלי של פונקציות טריגונומטריות,

פונקציות מעריכיות ולוגריתמיות

ופונקציות חזקה

$$- 2 \times 33\frac{1}{3} - 66\frac{2}{3} \text{ נקודות}$$

$$\text{סה"כ} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:

- (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
- (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
- (3) לטייטה יש להשתמש במחברת הבחינה. שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail. Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Sequences and Spatial Trigonometry ($33\frac{1}{3}$ points)

Answer one of the questions 1-2.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

Sequences

1. Given: an infinite geometric sequence in which all the terms are positive.

The third term of the sequence is 8 times greater than the sixth term of the sequence.

- א. How much greater is the sum of all the terms of the sequence than the sum of the terms located in the even positions?
- ב. The sum of the terms located in the odd positions is 2.
Calculate the value of the third term in the given sequence.

Spatial Trigonometry

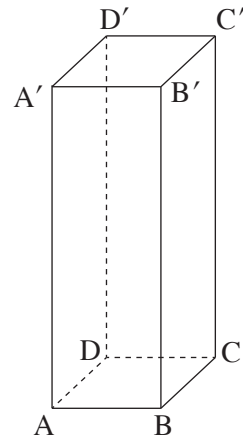
2. Given: a cuboid ABCDA'B'C'D', whose base, ABCD, is a square (see figure).

Also given: $AB = a$; $AA' = 3a$.

- א. (1) Express AC and AD' in terms of a.
(2) Explain why $AD' = CD'$.
- ב. Find the size of the angle AD'C.
- ג. Express the area of the triangle AD'C in terms of a.

D'E is a height of the triangle AD'C.

- ד. Find the size of the angle between D'E and the base of the cuboid, ABCD.



Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

($66\frac{2}{3}$ points)

Answer two of the questions 3-5 (each question — $33\frac{1}{3}$ points).

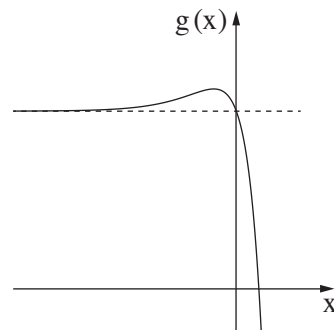
Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

3. Given: The function $f(x) = 3 \cdot \sin(x - \frac{\pi}{2})$ on the interval $-\pi \leq x \leq \pi$.
- א. (1) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes on the given interval.
 - (2) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$ on the given interval and determine what type of extrema they are.
 - ב. Sketch the graph of the function $f(x)$ on the given interval.
 - ג. Calculate the area of the region bounded by the graph of the function $f(x)$, the line $x = \pi$, and the x -axis, on the interval $\frac{\pi}{2} \leq x \leq \pi$.
4. Given: The function $f(x) = 4^{2x} - 4^x - 2$.
- א. (1) What is the domain [תחום ההגדרה] of the function $f(x)$?
 - (2) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes.
 - (3) Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$ and determine what type of extremum it is.

The figure to the right shows a sketch of the graph of the function $g(x) = -2f(x)$.

The function $g(x)$ has an asymptote whose equation is $y = 4$.

- ב. (1) What are the coordinates of the extremum [נקודת הקיצון] of the function $g(x)$?
- (2) What is the equation of the horizontal asymptote of the function $f(x)$? Explain.
- (3) Sketch the graph of the function $f(x)$.



5. Given: The function $f(x) = \frac{2\ln x + 3}{3}$.

- א. (1) What is the domain [תחום ההגדרה] of the function $f(x)$?
- (2) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes (if there are such points).
- (3) Find the intervals on which the function $f(x)$ increases and decreases (if there are such intervals).
- (4) Write the equation of the vertical asymptote of the function $f(x)$.
- (5) Sketch the graph of the function $f(x)$.
- ב. (1) Write the equations of the asymptotes of the derivative function $f'(x)$ that are perpendicular to the axes.
- (2) Sketch the graph of the derivative function $f'(x)$.

$b > 1$ is a parameter.

The area of the region bounded by the graph of the derivative function $f'(x)$, the x-axis, the line $x = 1$, and the line $x = b$ is equal to $\ln 4$.

- ג. Find the value of b .

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך