

Mathematics

5 Points – 2nd Exam

Instructions for examinees

א. Duration of the exam: Two hours and fifteen minutes

ב. Exam structure and breakdown of points:

This exam consists of two parts.

Part One: Analytical Geometry, Vectors,

Spatial Trigonometry, Complex Numbers

$$2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ points}$$

Part Two: Growth and Decay, Power Functions,

Exponential and Logarithmic Functions

$$1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ points}$$

$$\text{Total} \quad - \quad 100 \text{ points}$$

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

(3) For draft paper, use the pages in the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות לנבחן

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

$$2 \times 33 \frac{1}{3} - 66 \frac{2}{3} \text{ נקודות}$$

פרק שני: גדילה ודעיכה, פונקציות חזקה, פונקציות

מעריכיות ולוגריתמיות

$$1 \times 33 \frac{1}{3} - 33 \frac{1}{3} \text{ נקודות}$$

$$\text{סה"כ} \quad - \quad 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי

הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה מסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Geometry, and Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Given: Points A (0, 0), B (19, 0), and D (9, 0).

- א. Find the equation of the locus on which Points C are located for which CD is an angle bisector in Triangle ABC.
 - ב. What is the largest area of Triangle ABC that can be constructed as described in Item א?
 - ג. Find the coordinates of the two Points C for which the side BC in Triangle ABC is tangent to the locus whose equation you found in Item א.
- You may leave a square root sign in your answer.

2. $ABCA'B'C'$ is a right triangular prism. All edges of this prism are equal.

Let a be the length of the edge.

ABCK is a right pyramid. DK is a height of Pyramid ABCK, as shown in the figure.

Given: $DK = t \cdot AA'$,

the volume of Prism $ABCA'B'C'$ is 4.5 times greater
than the volume of Pyramid ABCK.

- א. Calculate the value of t .
- ב. Find the angle between the plane ABK and the plane ABC.

Given: The volume of pyramid ABCK is $12\sqrt{3}$.

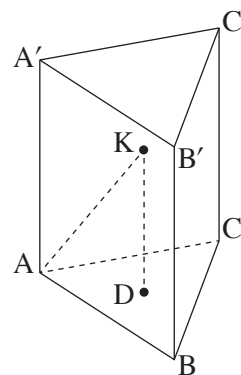
- ג. Find the value of a .

Given: Vertex A is at the origin of the axes, Vertex A' is on the positive side of the z-axis, and Vertex C is on the positive side of the y-axis.

The coordinates of Vertex B are positive.

- ד. (1) Find the coordinates of Vertex B' .
- (2) Find the equation of Plane $AB'K$.

You may leave a square root sign in your answers.



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3. א. Solve the equation $z^2 + (-5 + 2i)z + 7 + i = 0$.

Let w denote the solution of the equation in Item א that represents the point that is closer to the origin of the axes.

a_n is an arithmetic sequence. w is a term in the sequence and 1 is also a term in the sequence.

ב. (1) Explain why all the terms in this sequence have the form: $a_n = 1 + b \cdot i$. b is a real number.

(2) Explain why all the points on the Gauss plane that represent the terms of the sequence a_n , except for the point $(1, 0)$, are outside the unit circle.

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer one of the questions 4-5.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: The function $f(x) = \frac{e^x}{e^x + 1}$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the intervals where the function $f(x)$ is increasing and decreasing (if there are any).
- (3) Find the coordinates of the inflection points of the function $f(x)$ (if there are any).
- (4) Find the equation of the asymptotes of the function $f(x)$ that are perpendicular to the axes.
- (5) Sketch the graph of the function $f(x)$.

ב. Explain why $\int_a^{a+1} f(x) dx < 1$ is satisfied for any real number a . You may use a sketch.

ג. (1) $g(x)$ is a function that satisfies $f(x) = g(x) + \frac{1}{2}$.

Prove that the function $g(x)$ is an odd function.

(2) Explain why for every two numbers b and c which satisfy $0 < b < c$, the following is true:

$$\int_{-c}^{-b} f(x) dx + \int_b^c f(x) dx = c - b$$

In your answer, you may use a suitable sketch and apply symmetry considerations.

5. Given: The function $f(x) = \frac{(\ln x)^n}{\sqrt{x}}$. n is a natural number.

א. (1) Find the domain [תחום ההגדרה] of $f(x)$.

(2) Find the points of intersection of the function $f(x)$ with the axes (if there are any).

The area bounded by the graph of function $f(x)$, the x -axis, and the lines $x = 1$ and $x = e^2$ was rotated around the x -axis. The volume of the resulting solid of revolution was $\frac{32\pi}{2n+1}$.

ב. Find the value of n .

Insert the value of n that you found in Item ב in the function $f(x)$ and answer Items ג-ה.

ג. (1) Find the coordinates of the extrema of the function $f(x)$ and determine what type of extrema they are.

(2) Find the equation of the asymptote of the function $f(x)$ that is perpendicular to the x -axis.

The function $f(x)$ has an asymptote whose equation is $y = 0$.

ד. Sketch the graph of the function $f(x)$.

ה. The function $g(x)$ satisfies $g(x) = f(x) + m$; $m \neq 0$ is a parameter.

Given: There is a point at which the graph of the function $g(x)$ is tangent to the x -axis.

(1) Find the value of m .

(2) For which values of k does the equation $g(x) = k$ have a single solution?

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך