The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2018
Exam Number: 035581, 035806, 316
Appendix: Formula sheets for 5 points
English Translation (3)

Mathematics
5 Points — 1st Exam

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra and Probability
– 2 × 20 – 40 points
Part Two: Planar Geometry and Planar Trigonometry
– 1 × 20 – 20 points
Part Three: Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions
– 2 × 20 – 40 points
Total – 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
(3) For draft paper, use pages in the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשע"ח, 2018
מספר השאלון: 035581, 035806, 316
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה
5 יחידות לימוד - שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה והסתברות
2×20 — 40 נקודות
פרק שני: גאומטריה וטריגונומטריה במישור
1×20 — 20 נקודות
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות
2×20 — 40 נקודות
סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
(3) לטיוטה יש להשתמש במחברת הבחינה.
שימוש בטיוטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. A resort has two pools: Pool א and Pool ב.

The volume of Pool א is V_1 and the volume of Pool ב is V_2.

The pools are filled using 4 pipes of equal capacity.

One day, both pools were empty.

The pool attendants began to fill Pool א using the four pipes.

When Pool א was $\frac{1}{6}$ full, they transferred one of the pipes to Pool ב and began to fill the pool with that pipe.

When Pool א was half full, they transferred two more pipes to Pool ב.

Both pools became full at the same time.

Water flowed continuously through all the pipes until both pools were full.

Calculate the ratio $\frac{V_1}{V_2}$.

2. a_n is an arithmetic sequence whose common difference, d, is not 0.

Given: $a_7 = -a_{17}$.

א. Find a_{12}.

ב. **(1)** Does the sequence contain a term whose value equals $(-a_1)$? Explain.

(2) Find a natural number n such that the sum of the first n terms in the sequence equals 0.

ג. Is there a natural number n such that $a_n \cdot a_{n+1} < 0$? If so, find such an n.
If not, explain.

ד. Is it possible to know how many negative terms there are in the sequence?
Explain (distinguish between different cases).

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3. Michal has a balanced die.

The number 2 is written on three faces of the die, and the number 4 is written on the other three faces.

Galit has a different balanced die. The number 1 or the number 3 is written on each face of Galit's die.

Michal and Galit play a game with five rounds. The player who wins more rounds than the other wins the game. In each round of the game, each of them throws her die once.

The winner of the round is the player whose die shows a higher number.

Given: On a single round, the probability of Michal defeating Galit is $\frac{7}{12}$.

א. On how many faces of Galit's die is the number 1 written? Explain your answer.

ב. What is the probability that Galit will win the game?

ג. What is the probability that Galit will win the game, if we know that she won the first round?

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Part Two — Planar Geometry and Planar Trigonometry

(20 points)

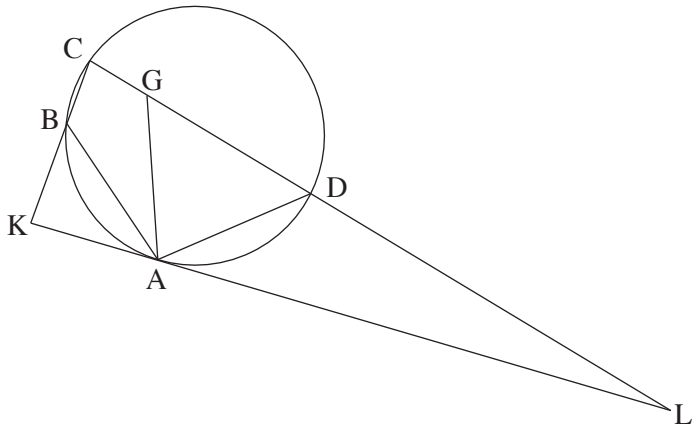
Answer one of the questions 4-5.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Quadrilateral ABCD is inscribed in a circle.

Point G is located on Side CD such that $AB = AG$ and also $CB = CG$.

The tangent to the circle at Point A intersects the extension of Side CD at Point L and intersects the extension of Side CB at Point K (see figure).



א. Prove that $AD = AG$.

ב. **(1)** Prove that $\triangle ABK \sim \triangle CDA$.

(2) Prove that $AD^2 = BK \cdot CD$.

ג. Show that $\frac{S_{\triangle LDA}}{S_{\triangle KAB}} = \frac{LA}{AK}$.

5. Given: a parallelogram ABCD. AC is the long diagonal, as shown in the figure.

A circle whose center is O is inscribed in Triangle ABC.

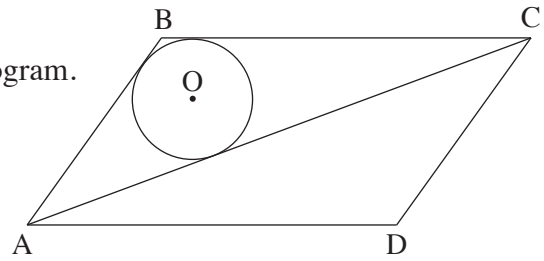
Given: Point O is located at distances of 6 and 3 from the lines AD and AC, respectively;

$OA = 10$.

א. Calculate the sizes of the angles of the parallelogram.

ב. Calculate the length of the diagonal AC.

ג. Calculate the area of the parallelogram.



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Part Three—Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions (40 points)

Answer <u>two</u> of the questions 6-8 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The functions $f(x)=\frac{\sin x}{\sqrt{\cos x}}$, $g(x)=\frac{\cos x}{\sqrt{\sin x}}$.

Answer Item א for the interval $-\frac{\pi}{2}\le x\le\pi$.

א. **(1)** Find the domain [תחום ההגדרה] of the function f(x).

(2) Find the equations of the asymptotes of the function f(x) that are perpendicular to the x-axis.

(3) Find the intervals on which the function f(x) is increasing and decreasing (if there are such intervals).

(4) Sketch the graph of the function f(x).

Answer Item ב for the interval $-\frac{\pi}{2}\le x\le\pi$, too.

ב. **(1)** Find the domain [תחום ההגדרה] of the function g(x).

(2) Prove: $g(x)=-f\left(x-\frac{\pi}{2}\right)$.

(2) Sketch the graph of the function g(x).

You may make use of your answers to the previous items.

ג. Find the value of the expression $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(x)\,dx$. Explain your answer.

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7. Given: The family of functions $f(x)=\frac{(x-2)^2}{x^2-a}$. a is a parameter, $a\neq 0$, $a\neq 4$.

Answer Item א. Express in terms of a, if necessary. Distinguish between $a>0$ and $a<0$, if necessary.

א. **(1)** Find the domain [תחום ההגדרה] of the function f(x).

(2) Find the coordinates of the points at which the graph of the function f(x) intersects the axes.

(3) Find the equation of the asymptote of the function f(x) that is parallel to the x-axis.

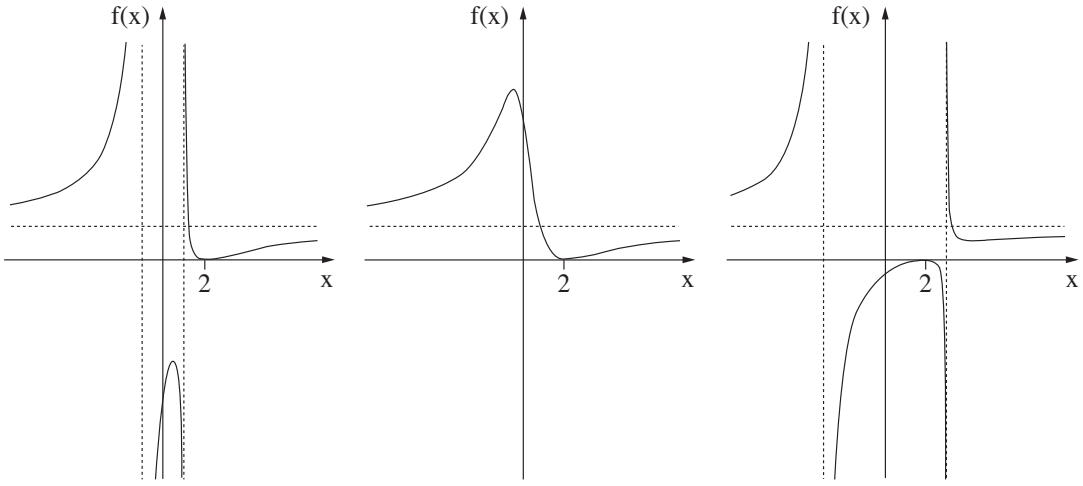
(4) Find the equations of the asymptotes of the function f(x) that are perpendicular to the x-axis (if there are any).

Answer Item ב. Express in terms of a, if necessary. Distinguish between $a>4$ and $a<4$, if necessary.

ב. Find the coordinates of the extrema [נקודות הקיצון] of the function f(x) and determine what kind of extrema they are.

ג. Three possible graphs of the function f(x) are shown below, each for a different value of a.

Specify the range of values [תחום הערכים] of a that corresponds to each of the graphs I-III. Explain your answer.



III II I

Continued on page 7

8. Given: the function $f(x)=\frac{1}{x^3}$.

A line was drawn that is tangent to the function f(x) at the point where x = t.

Given: $1 \le t \le 5$.

The tangent intersects the x-axis at Point A and the y-axis at Point B.

Point O is the origin of the axes.

א. Find the x-coordinate of the point of tangency for which the sum of the legs [הניצבים] of Triangle AOB is minimal (smallest).

ב. Find the x-coordinate of the point of tangency for which the sum of the legs [הניצבים] of Triangle AOB is maximal (largest).

Good Luck!

בהצלחה!