

Mathematics

3 Points — 3rd Exam

Instructions for examinees

א. Duration of the exam: Two hours

ב. Exam structure and breakdown of points:

This exam has six questions in the topic areas:

Algebra, Differential and Integral Calculus.

You must answer four questions — $4 \times 25 = 100$ points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to disqualification of your exam.

(3) For draft paper, use the answer booklet.

Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שעותיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות — $25 \times 4 = 100$ נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-6 (each question – 25 points).

Note! If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. The price of a table is 2 times greater than the price of a chair.

In a sale, the price of the table was discounted by 15%, and the price of the chair was discounted by 25%.

Eli bought one table and 3 chairs at the sale prices and paid a total of 1,343 shekels.

- א. Calculate what the price of a chair was before the sale, and what the price of a table was before the sale.

Eli's budget would have allowed him to buy exactly one table and 3 chairs at the pre-sale prices.

- ב. Is the amount of money that Eli saved in the sale enough to buy another chair? Explain.

2. ABCD is a square. Vertex A is located on the y-axis (see figure).

Given: The x-coordinate of Vertex C is 24,
the equation of Diagonal AC is $y = \frac{3}{4}x + 4$.

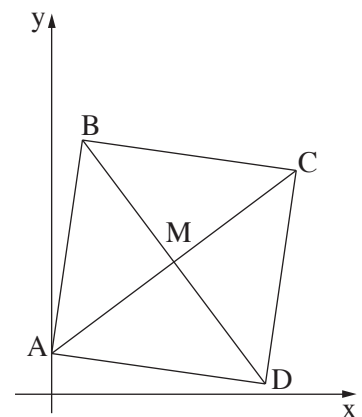
- א. (1) What are the coordinates of Vertex A?
(2) Find the y-coordinate of Vertex C.

M is the intersection point of the diagonals in Square ABCD.

- ב. (1) What is the slope of Diagonal BD?
(2) Find the equation of Diagonal BD.

The line BD intersects the y-axis at Point E.

- ג. Find the perimeter of Triangle AME.



3. Given: a circle whose equation is $(x + 4)^2 + (y + 2)^2 = 40$.

Point A is the point where the circle intersects the positive part of the x-axis (see figure).

א. Find the coordinates of Point A.

Given: Point B $(-6, 4)$.

ב. Show that Point B is located on the circle.

Point C is located on the circle such that AC is a diameter of the circle.

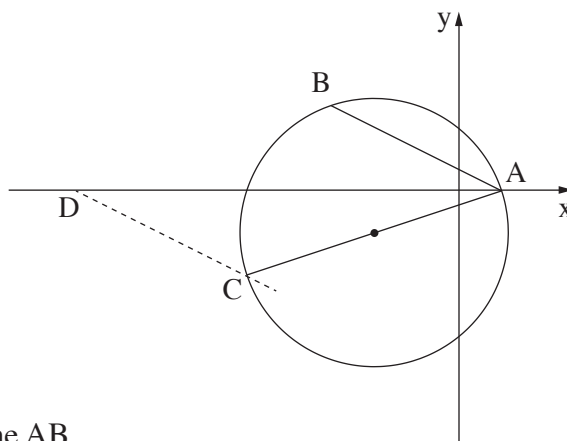
ג. Find the coordinates of Point C.

A line was drawn through Point C, parallel to Line AB.

ד. Find the equation of this line (the dashed line in the figure).

The line whose equation you found in Item ד intersects the x-axis at Point D.

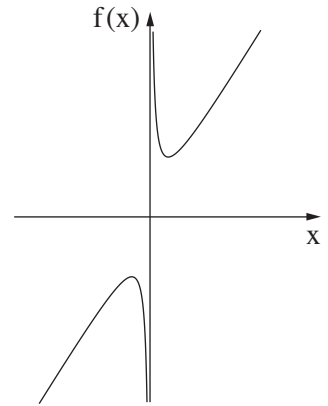
ה. Calculate the area of Triangle ADC.



Differential and Integral Calculus

4. Shown at right is the graph of the function $f(x) = 4x + \frac{16}{x}$.

- א. What is the domain [תחום ההגדרה] of the function $f(x)$?
- ב. Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$, and determine what kinds of extrema they are, based on the graph.



A tangent was drawn to the graph of the function $f(x)$ at the point where $x = 4$.

- ג. (1) Find the slope of the tangent.
- (2) Find the equation of the tangent.
- ד. (1) Find the equation of the tangent to the graph of the function $f(x)$ at the function's maximum point.
- (2) Find the coordinates of the point at which the two tangents intersect each other.

5. Shown at right is a diagram of the graph of the function $f(x) = -x^3 + 3x^2 + 16$.

Point A is the point of intersection of the graph of the function $f(x)$ with the y-axis.

- א. Find the coordinates of Point A.

A line parallel to the x-axis was drawn through Point A.

- ב. Find the equation of this line.

The line intersects the graph of the function $f(x)$ at an additional point, B.

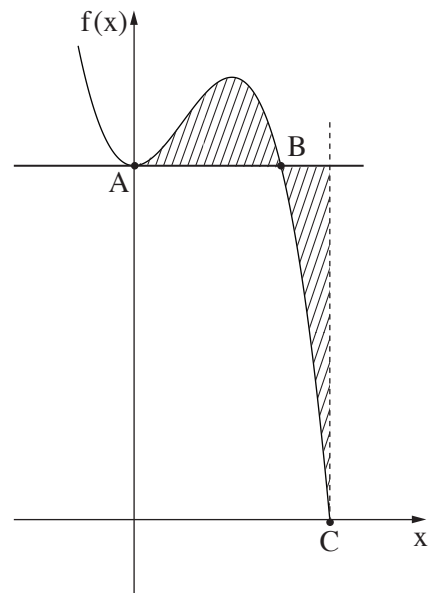
(The y-coordinate of Point B is the same as the y-coordinate of Point A.)

- ג. Find the coordinates of Point B.

The point of intersection of the graph of the function $f(x)$ with the x-axis is $C(4, 0)$.

A line perpendicular to the x-axis was drawn through Point C (the dashed line in the figure).

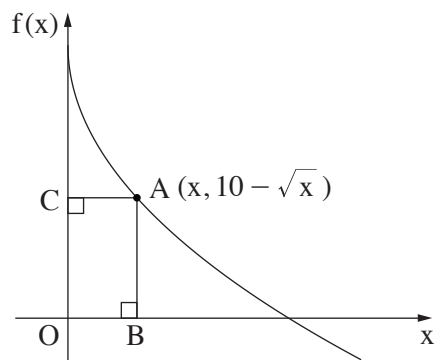
- ד. Calculate the area of the shaded region in the figure:
the region bounded by the graph of the function $f(x)$, the line AB, and the line perpendicular to the x-axis.



6. Point A is located in the first quadrant, on the graph of the function $f(x) = 10 - \sqrt{x}$, ($x > 0$). Perpendiculars are drawn from Point A to the axes, intersecting the axes at Points B and C, as shown in the figure.

O is the origin of the axes.

- א. What are the coordinates of Point A for which the perimeter of Rectangle ABOC is minimal (smallest)?
ב. What is the minimum perimeter of Rectangle ABOC?



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך