The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2017, ***Moed Bet***
Exam Number: 035804, 314
Appendix: Formula sheets for 4 points
English Translation (3)

Mathematics

4 Points — First Questionnaire

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra, Analytical Geometry, Probability – 2×20 — 40 points
Part Two: Planar Geometry and Planar Trigonometry – 1×20 — 20 points
Part Three: Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions – 2×20 — 40 points
Total — 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached)
(3) A Hebrew-foreign language / foreign language-Hebrew dictionary

ד. Special instructions:
(1) Do not copy the question; only indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
(3) For draft paper, use the pages in the answer booklet. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשע"ז, **מועד ב**
מספר השאלון: 035804, 314
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות
2×20 — 40 נקודות
פרק שני: גאומטריה וטריגונומטריה במישור
1×20 — 20 נקודות
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש
2×20 — 40 נקודות
סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
(3) לטיוטה יש להשתמש במחברת הבחינה.
שימוש בטיוטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain <u>all</u> of your work, including calculations, clearly and in detail. Lack of detail may lower your score or lead to the disqualification of your exam.

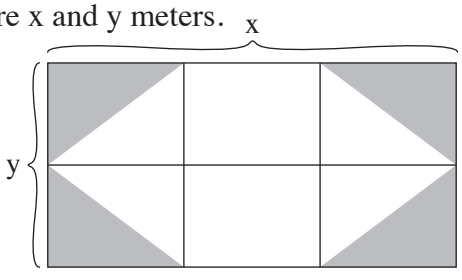
Part One — Algebra, Analytical Geometry, and Probability

(40 points)

Answer <u>two</u> of the questions 1-3 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. The Dotan family decided to pave their garden, which is rectangular.
The lengths of the sides of the garden are x and y meters.
The garden was divided into 6 identical rectangles.
In each of the corner rectangles, a right triangle was inscribed [חסום].
The legs [ניצבים] of each of these triangles coincided with the sides of the rectangle.
The area of each triangle was paved with gray stone, as shown in the figure.
The remaining area of the garden was paved in white stone.



א. (1) Express the area of the garden that was paved with gray stone in terms of x and y.

(2) Express the area of the garden that was paved with white stone in terms of x and y.

The cost of paving with gray stone is 75 shekels per m^2.
The cost of paving with white stone is 60 shekels per m^2.
Given: One side of the garden is 3 meters longer than the other side.
The cost of paving the entire garden was 1,170 shekels.

ב. Find the lengths of the garden's sides.

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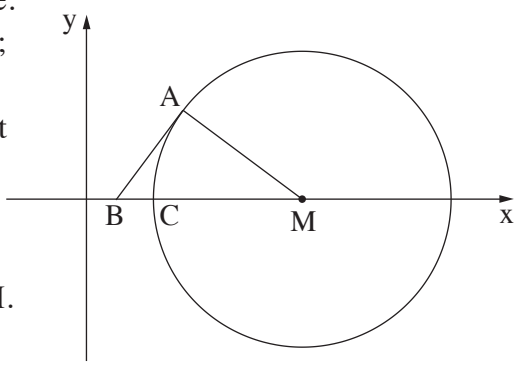
2. The figure to the right shows a circle.
Given: The radius of the circle is 20;
the center of the circle, M,
is located on the positive part
of the x-axis;
Point A (13,12) is located
on the circle.



א. Find the coordinates of Point M.

A tangent to the circle was drawn through Point A. This tangent intersects the x-axis at Point B.

ב. Find the coordinates of Point B.

ג. Find the equation of the circle that circumscribes [חוסם] Triangle BAM.

C is the point at which the given circle intersects the x-axis, as shown in the figure.

ד. (1) Find the x-coordinate of Point C.

(2) For which values of k does the line x = k intersect the two circles (and is not tangent to either of them)?

3. In a certain city, a survey was conducted to examine whether teenage boys and girls engage in physical exercise. The number of boys included in the survey was twice the number of girls included in the survey.
The survey showed that $\frac{3}{4}$ of the girls who participated in the survey engage in physical exercise and $\frac{4}{5}$ of the boys who participated in the survey engage in physical exercise.

א. An individual was randomly chosen from among all the survey participants (boys and girls).
What is the probability that the individual who was chosen engages in physical exercise?

ב. An individual was randomly chosen from among all the survey participants, and it was found that that individual engages in physical exercise.
What is the probability that a girl was chosen?

ג. 4 of the survey participants were randomly chosen.
What is the probability that at least 2 of the participants who were chosen were girls who engage in physical exercise?

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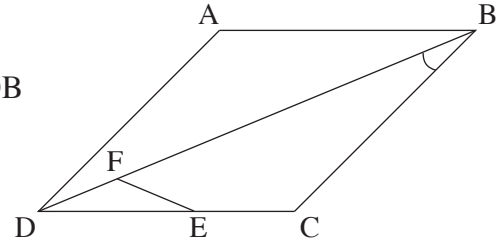
Part Two — Planar Geometry and Planar Trigonometry

(20 points)

Answer <u>one</u> of the questions 4-5.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. ABCD is a rhombus.
Point E is located on Side DC and Point F is located on Diagonal DB (see figure).
Given: Quadrilateral BCEF can be circumscribed by a circle.

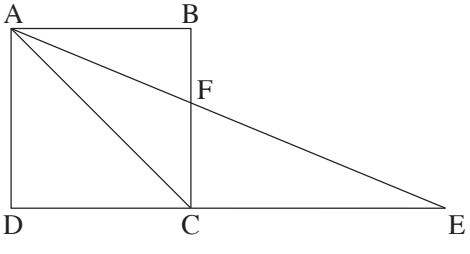


א. (1) Prove that $\sphericalangle FED = \sphericalangle CBD$.
(2) Prove that Triangle DFE is an isosceles triangle.

ב. Prove that $\Delta DFE \sim \Delta DCB$.

ג. Given: DB = 3DE; the area of Triangle DFE is 2 cm^2.
Calculate the area of Rhombus ABCD.

5. Given: Square ABCD.
Point E is located on the extension of Side DC, as shown in the figure.
Triangle ACE is an isosceles triangle (AC = CE).
The line AE intersects Side BC at Point F.



א. Find the angles of Triangle ACE.
The area of Triangle ACE is $8\sqrt{2}$ cm^2.

ב. Calculate the length of one side of the square.

ג. Calculate the length of Segment DF.

ד. Calculate the length of the radius of the circle that circumscribes Triangle DFE.

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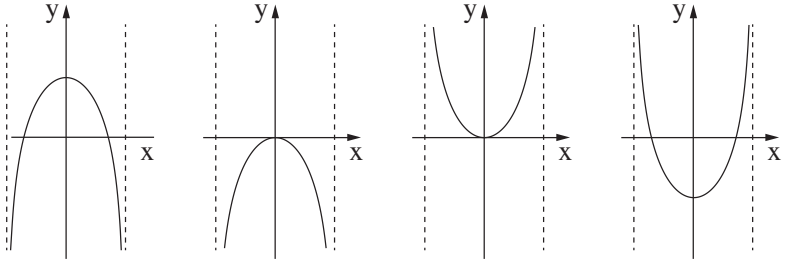
Part Three — Differential and Integral Calculus of Polynomials, Rational Functions and Root Functions (40 points)

Answer <u>two</u> of the questions 6-8 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x)=\frac{5}{(2x-4)^2}$.
   - א. Find the domain [תחום ההגדרה] of the function f(x).
   - ב. Find the asymptotes of the function f(x) that are perpendicular to the axes.
   - ג. Find the intervals on which the function f(x) increases and decreases.
   - ד. Sketch the graph of the function f(x).
   - ה. (1) Find the asymptotes of the function $-f(x)$ that are perpendicular to the axes.
     (2) Sketch the graph of the function $-f(x)$.

7. Given: The function $f(x)=x\sqrt{4-x^2}$.
   - א. Find the domain [תחום ההגדרה] of the function f(x).
   - ב. (1) Find the coordinates of the points at which the graph of the function f(x) intersects the axes.
     (2) Find the coordinates of the extrema [נקודות הקיצון] of function f(x) and determine what type of extrema they are.
   - ג. Sketch the graph of the function f(x).
   - ד. Which of the graphs (I-IV) shown at the end of the question is the graph of the function f'(x)? Explain.
   - ה. Calculate the area of the region that is bounded by the function f'(x), the x-axis, the y-axis, and the line x = 1.



IV III II I

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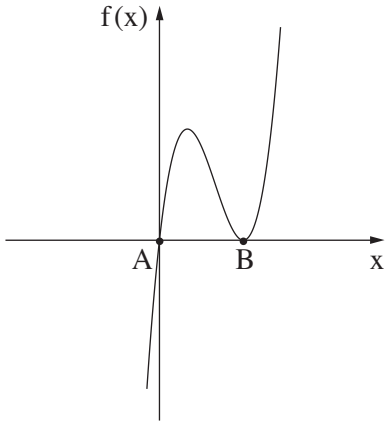
8. Below is a sketch of the graph of the function $f(x) = x^3 - 6x^2 + 9x$.



א. Find the coordinates of Points A and B, the points at which the graph of the function f(x) intersects the x-axis.

Point C is located on the graph of the function f(x).
Given: $x_A < x_C < x_B$.
(The x-coordinate of Point C lies between the x-coordinate of Point A and the x-coordinate of Point B.)

ב. Find the coordinates of Point C for which the area of Triangle ABC will be greatest [מקסימלי].

ג. Is Point C an extremum [נקודת קיצון] of the function f(x)? Explain.

Good Luck!

בהצלחה!