

**The State of Israel
Ministry of Education**

Exam Type: *Bagrut*

Exam Date: Summer 2017, *Moed Bet*

Exam Number: **035581**

Appendix: Formula sheets for 5 points

English Translation (3)

מדינת ישראל

משרד החינוך

סוג הבחינה: בגרות

מועד הבחינה: קיץ תשע"ז, מועד ב

מספר השאלון: 035581

נספח: דפי נוסחאות ל-5 יחידות לימוד

תרגום לאנגלית (3)

**Mathematics
Reform for Meaningful
Learning Program
First Questionnaire — 5 Points**

**מתמטיקה
על פי הרפורמה ללמידה משמעותית
שאלון ראשון מ-5 יחידות לימוד**

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra and Probability

– $2 \times 20 = 40$ points

Part Two: Planar Geometry and Planar

Trigonometry – $1 \times 20 = 20$ points

Part Three: Differential and Integral Calculus

of Polynomials, Root Functions, Rational

Functions, and Trigonometric Functions

– $2 \times 20 = 40$ points

Total – 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

(3) For draft paper, use pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of your exam.

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה והסתברות

$20 \times 2 = 40$ נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

$20 \times 1 = 20$ נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של

פולינומים, של פונקציות שורש, של פונקציות

רציונליות ושל פונקציות טריגונומטריות

$20 \times 2 = 40$ נקודות

סה"כ – 100 נקודות

ה. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות

התכנות במחשבון הניתן לתכנות. שימוש

במחשבון גרפי או באפשרויות התכנות במחשבון

עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום

במחברת את שלבי הפתרון, גם כאשר

החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט

ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת

הבחינה.

(3) לטיטה יש להשתמש במחברת הבחינה או

בדפים שקיבלת מהמנשיחים. שימוש בטיטה

אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

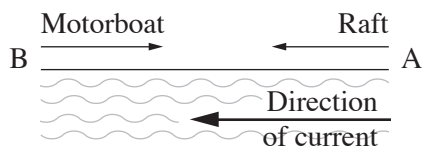
Note! Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Towns A and B are located on the bank of a river that flows at a constant speed. The direction of the current is from A to B.



A motorboat left Town B in the direction of Town A. The boat sailed against the direction of the current. At the same time, a raft left Town A in the direction of Town B. The raft sailed in the direction of the current.

The speed of the motorboat in still water is constant and is 4 times greater than the speed of the river current. The speed of the raft in still water is zero. In flowing water, the raft sails with the current.

The boat and the raft met 3 hours and 45 minutes after they set off, and then continued on their way. The motorboat reached Town A and immediately turned around and traveled back to Town B. When the motorboat reached Town B, the raft was at a distance of 35 km from Town B.

- א. Calculate the speed of the current and the speed of the motorboat in still water.
- ב. On the trip back to Town B, the motorboat met the raft a second time. How much time passed since the raft left Town A till the motorboat and the raft met a second time?

/Continued on page 3/

2. Given: a general sequence [סדרה כללית] a_n .

Let S_n denote the sum of the first n terms in the sequence a_n .

Given: $S_n = k - \frac{1}{3^{n+1}}$ for every natural number n . k is a constant.

א. Express a_1 and the general term a_n for $n > 1$ in terms of n and k ,
if necessary.

ב. Find the value of k for which a_n is a geometric sequence.
Justify your answer.

We define: $T = a_2^2 + a_5^2 + a_8^2 + \dots$ (the sum of the square of every third term of
the sequence a_n starting with a_2).

ג. Calculate T .

3. In Box I, there are 10 balls, some blue and the rest red, and in Box II, there are 7 blue balls and 3 red balls. A ball is taken at random from Box I. If it is red, we move it to Box II. If it is blue, we return it to Box I. Again, a ball is taken at random from Box I, and again, if it is red, we move it to Box II, and if it is blue, we return it to Box I. Afterwards, one ball is taken out of Box II at random.

- א. Given: the probability that after the two times balls were taken from Box I, only one red ball will be moved from Box I to Box II is $\frac{19}{36}$. Calculate the number of blue balls that were in Box I before the first ball was taken out.

Answer Items ב and ג for the number of blue balls that you calculated in Item א.

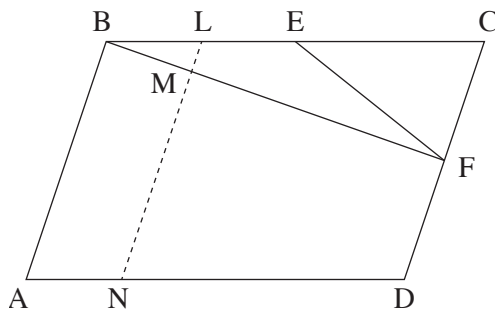
- ב. What is the probability that the ball that was taken out of Box II is red?
ג. It is known that the ball taken from Box II is red. What is the probability that after taking the red ball from Box II, exactly three red balls remained in it?

Part Two — Planar Geometry and Planar Trigonometry (20 points)

Answer one of the questions 4-5.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Quadrilateral ABCD is a parallelogram.
Angle A is an acute angle.
Point E is the midpoint of Side BC.
Point F is the midpoint of Side CD
(see figure).



- א. The area of Triangle ECF is S .
Express the area of the parallelogram ABCD in terms of S . Justify your answer.
- ב. Point L is the midpoint of Segment BE.
A line was drawn through Point L, parallel to AB.
This line intersects BF and AD at Points M and N, respectively.
Calculate the ratio $\frac{LM}{MN}$.
- ג. Given: $BE = EF$.
Can Quadrilateral ABFD be circumscribed by a circle [להיחסם במעגל]?
Justify your answer.
5. Trapezoid ABCD ($AB \parallel DC$) is circumscribed by a circle [חסום במעגל].
Given: $AB = a$, $CD = b$ ($a < b$).
 $\angle C = 60^\circ$.
- א. Express the legs of the trapezoid [שוקי הטרפז], AD and BC, in terms of a and b .
- Given: $a = 4$, the length of the diagonal BD is $4\sqrt{7}$.
- ב. Calculate the value of b .
- ג. (1) R is the radius of the circle circumscribing the trapezoid. Find R.
(2) Explain why it is possible to inscribe a circle in Trapezoid ABCD.
(3) r is the radius of the circle inscribed in the trapezoid. Find r .

Part Three — Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions and Trigonometric Functions

(40 points)

Answer two of the questions 6-8 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x) = a - \frac{2}{x-2} + \frac{1}{(x-2)^2}$. a is a parameter.

Answer Item א. Express your answers in terms of a , if necessary.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
(2) Find the equations of the asymptotes that are perpendicular to the axes.
(3) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$ (if there are any) and determine what type of extrema they are.
(4) Find the intervals on which the function $f(x)$ increases and decreases [תחומי העלייה והירידה של הפונקציה].

Given: the graph of the function $f(x)$ is tangent to the x -axis.

- ב. Find the value of a .

Insert the value of a that you found and answer Items ג and ד.

- ג. Sketch the graph of the function $f(x)$.

- ד. Given: The function $g(x) = |f(x) + k|$.

It is known that the graph of the function $g(x)$ is tangent to the horizontal asymptote of the graph of the function $f(x)$.

Find k (find both possibilities). Justify your answer.

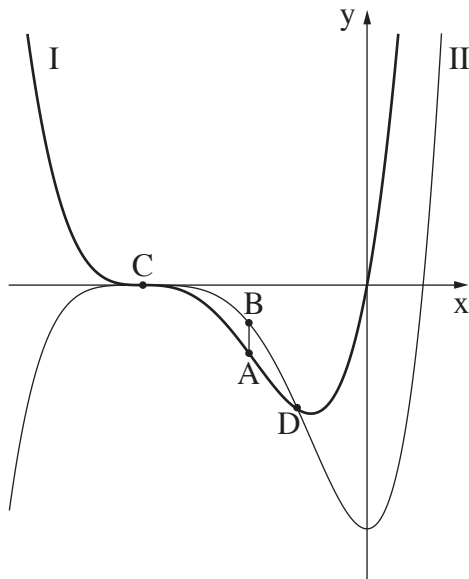
7. Shown to the right are graphs of the functions $f(x)$ and $f'(x)$.

א. Which graph, I or II, represents $f(x)$ and which represents $f'(x)$? Justify your answer.

Given: The function $f'(x) = x(x + b)^3$.

$b > 1$ is a parameter.

The graph of the function $f(x)$ has an inflection point [נקודת פיתול] at $x = -1$.



- ב. Find the value of b .

C and D are the intersection points of the functions $f(x)$ and $f'(x)$ on the interval $x < 0$, as shown in the figure.

Points A and B are located on graphs I and II, respectively, such that the line AB is perpendicular to the x -axis.

Given: $x_C < x_A < x_D$,

$$x_C = -4,$$

$$x_D = 1 - \sqrt{5}.$$

- ג. Find the x -coordinate of Points A and B for which the length of the segment AB is maximal.

(It is possible to answer this item without finding the function $f(x)$).

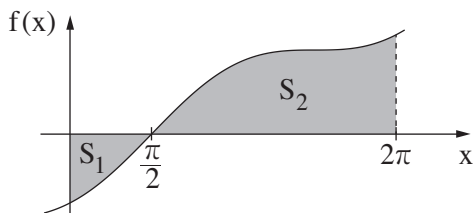
8. $f(x)$ is a function that is defined for all values of x .

The graph of the function $f(x)$ intersects the y -axis on its negative part.

The only intersection point of the graph of the function $f(x)$ with the x -axis is $(\frac{\pi}{2}, 0)$ (see figure).

Given: The area of the region bounded by the graph of the function $f(x)$, the axes, and the line $x = 2\pi$ (the gray area in the figure) is $10\pi^2 + 16$.

Also given: $\int_0^{2\pi} f(x) dx = 8\pi^2$.



א. Find the area of the region bounded by the graph of the function $f(x)$ and the axes (the area labeled S_1 in the figure).

The function $F(x)$ is a primitive function [פונקציה קדומה] of the function $f(x)$.

Given: $F(0) = 0$.

ב. Find $F(\frac{\pi}{2})$.

Given: $f'(x) = 8 \sin x + 8$.

ג. Find $f(x)$.

Good Luck!

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 Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך