The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2017
Exam Number: 035807, 317
Appendix: Formula sheets for 5 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשע"ז, 2017
מספר השאלון: 317, 035807
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

Mathematics

5 Points – Second Questionnaire

Instructions for examinees

א. Duration of the exam: Two hours and fifteen minutes

ב. Exam structure and breakdown of points:

This exam has two parts.

Part One: Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

$2\times 33\frac{1}{3}$ — $66\frac{2}{3}$ points

Part Two: Growth and Decay, Exponential and Logarithmic Functions

$1\times 33\frac{1}{3}$ — $33\frac{1}{3}$ points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/ foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
(3) For draft paper, use the pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים.

פרק ראשון: גאומטריה אנליטית, וקטורים, טריגונומטריה במרחב, מספרים מרוכבים

$2\times 33\frac{1}{3}$ — $66\frac{2}{3}$ נקודות

פרק שני: גדילה ודעיכה, פונקציות מעריכיות ולוגריתמיות

$1\times 33\frac{1}{3}$ — $33\frac{1}{3}$ נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
(3) לטיוטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים. שימוש בטיוטה אחרת עלול לגרום לפסילת הבחינה.

ב ה צ ל ח ה !

Questions

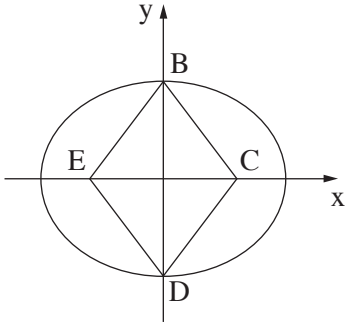
Note! Explain all of your work, including calculations, clearly and in detail. Lack of detail may lower your score or lead to the disqualification of your exam.

Part One – Analytical Geometry, Vectors, Spatial Geometry and Complex Numbers ($66\frac{2}{3}$ points)

Answer two of the questions 1-3 (each question – $33\frac{1}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Given: Rhombus BCDE
Vertices B and D are located on the y-axis, and
Vertices C and E are located on the x-axis.
Given: The length of a side of the rhombus is 5,
the length of its height is 4.8, and
the length of Diagonal BD is greater than the length of Diagonal CE.



A canonical ellipse [אליפסה קנונית], whose foci [מוקדים] are Points C and E, passes through Vertices B and D (see figure).

א. (1) Find the coordinates of the vertices of the rhombus.

(2) Find the equation of the ellipse.

A parabola, whose equation is $y^2 = 2px$, intersects the ellipse in the first quadrant at Point M.

Given: The y-coordinate of M is $\sqrt{15}$.

ב. Prove that the focus of the parabola is located at Point C.

ג. A line that is parallel to the y-axis is drawn through Point E.

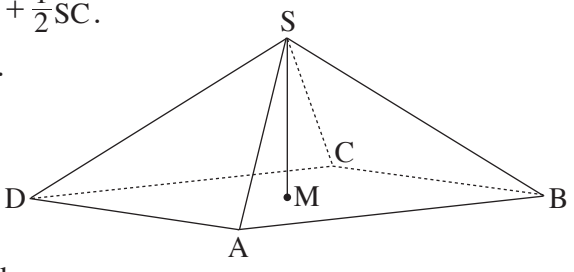
P is a point on the parabola that is located at a distance of k from this line.

Find the ratio $\frac{PC}{k}$. Explain.

Continued on page 3 ▶

2. Given: the right pyramid [פירמידה ישרה] SABCD, whose base, ABCD, is a square.

M is a point such that $\overrightarrow{SM} = \frac{1}{2}\overrightarrow{SA} + \frac{1}{2}\overrightarrow{SC}$.



א. (1) Prove that $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC}$.

(2) Prove that $\overrightarrow{SM}$ is perpendicular to $\overrightarrow{AC}$.

(3) Explain why SM is a height of the pyramid.

Given: $A(\sqrt{3}, 1, 0)$, $C(-\sqrt{3}, -1, 0)$, Points B and D are located on the plane $z = 0$, and the volume of Pyramid SABCD is 16.

ב. (1) Find the coordinates of Point M.

(2) Find the coordinates of Vertex S (find both possibilities).

Let S_1 and S_2 denote the points that you found for Sub-item ב (2).

ג. (1) Find the equation of the plane AS_1S_2.

(2) Is Point C located on the plane AS_1S_2? Explain.

Continued on page 4 ▶

3. א. Find the complex numbers z for which $z^3 = -1$.

Let z_1, z_2 and z_3 denote the solutions for the equation in Item א.
Given: z_2 is real.

ב. (1) Show that z_1, z_2 and z_3 are three consecutive terms in a geometric sequence.

(2) z_1, z_2 and z_3 are the first three terms in the geometric sequence z_n.
Find z_5, the fifth term in the sequence.

ג. (1) z_{13}, z_{14} and z_{15} (the 13th, 14th and 15th terms in the sequence z_n that you found for Item ב) are represented by Points A, B and C, respectively, on the Gaussian plane.
Calculate the area of Triangle ABC.

(2) K, L and M are three points on the Gaussian plane that represent three consecutive terms in the sequence z_n.
Explain why Triangle KLM is congruent to the triangle whose area you found for Sub-item ג(1).

Continued on page 5 ▶

Part Two — Growth and Decay, Exponential and Logarithmic Functions ($33\frac{1}{3}$ points)

Answer <u>one</u> of the questions 4-5.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: The function $f(x)=\frac{e^{x^2}-2x}{e^{x^2}}$.

א. (1) Find the domain [תחום ההגדרה] of the function f(x).

(2) Find the extrema [נקודות הקיצון] of the function f(x) and determine what type of extrema they are.

(3) Find the intervals on which the function f(x) increases and decreases.

(4) Find the asymptotes of the function f(x) that are perpendicular to the axes (if there are any).

(5) Sketch a graph of the function f(x).

Given: The function $g(x)=\frac{1}{f(x)}$.

Refer to your answers to Item א when you answer Item ב.

ב. (1) Explain why the function g(x) is defined for every x.

(2) What are the coordinates of the extrema of function g(x) and what type of extrema are they? Explain your answer.

(3) Find the intervals on which the function g(x) increases and decreases.

(4) Find the asymptotes of the function g(x) that are perpendicular to the axes (if there are any). Explain your answer.

(5) Add a sketch of the graph of the function g(x) to your sketch of the graph of function f(x).

Continued on page 6 ◀

5. Given: The function $h(x)=\frac{x+3}{x}$.

א. Find the domain [תחום ההגדרה] of h(x).

ב. Find the interval on which $h(x)>0$.

On the interval on which $h(x)>0$, the function f(x), for which $f'(x)=\frac{h'(x)}{h(x)}$, is given.

Also given: The graph of the function f(x) passes through the point $(3,\ell n2)$.

We know that the function f(x) has one horizontal asymptote.

ג. Find the function f(x).

ד. Find the asymptotes of the function f(x) that are perpendicular to the axes.

ה. Find the intervals on which the function f(x) increases and decreases (if there are any).

ו. Sketch the graph of the function f(x).

Good Luck!

בהצלחה!