

Mathematics

5 Points — First Questionnaire

Instructions for examinees

- א. Duration of the exam: Three and a half hours
- ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra and Probability
 $2 \times 20 = 40$ points
Part Two: Planar Geometry and Planar Trigonometry
 $1 \times 20 = 20$ points
Part Three: Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions
 $2 \times 20 = 40$ points
Total — 100 points
- ג. Material that may be used during the exam:
 - (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
 - (2) Formula sheets (attached).
 - (3) A Hebrew-foreign language/foreign language-Hebrew dictionary.
- ד. Special instructions:
 - (1) Do not copy the question, but do indicate the number of the question you are answering.
 - (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.
 - (3) For draft paper, use pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of your exam.

Good Luck!

מתמטיקה

5 יחידות לימוד - שאלון ראשון

הוראות לנבחן

- א. משך הבחינה: שלוש שעות וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה והסתברות
 $20 \times 2 = 40$ נקודות
פרק שני: גאומטריה וטריגונומטריה
במישור $20 \times 1 = 20$ נקודות
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות
 $20 \times 2 = 40$ נקודות
סה"כ — 100 נקודות
- ג. חומר עזר מותר בשימוש:
 - (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
 - (2) דפי נוסחאות (מצורפים).
 - (3) מילון עברי-לועזי/לועזי-עברי.
- ד. הוראות מיוחדות:
 - (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
 - (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
 - (3) לטייטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים. שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Noga cycled along a trail of a certain length at four different constant speeds.

Each time she completed a section that was one quarter of the length of the trail, she increased her speed and cycled at a speed that was twice the magnitude of her previous speed.

In the last section, she cycled at a speed of 40 km/h.

Noga set off at 8:00 am and finished the trail at 11:45 am.

א. What was the length of the trail?

- ב. Daniel set off on the same trail at 9:45 am and traveled at a constant speed for the entire length of the trail.

He also reached the end of the trail at 11:45 am.

At which of the four sections of the trail did Daniel catch up with Noga for the first time, and at what time?

2. Given: The sequence $a_n = \frac{(2^n + 1)(2^n - 1)}{2^n}$.

b_n and c_n are geometric sequences in which all the terms are positive. For every natural n in these sequences $a_n = b_n - c_n$.

Given: $c_3 = \frac{1}{8}$; $b_6 = 64$.

א. (1) Find b_1 and the common ratio [מנה] of the sequence b_n .

(2) Find c_1 and the common ratio of the sequence c_n .

Let A_n denote the sum of the first n terms in the sequence a_n .

Let B_n denote the sum of the first n terms in the sequence b_n .

Let C_n denote the sum of the first n terms in the sequence c_n .

ב. Show that $C_n = B_n - A_n$.

ג. For what values of n is the inequality $0.9 < B_n - A_n < 1$ true?

3. In a large retirement home, some of the residents have a mobility scooter and the rest do not.

If we randomly choose 9 residents from this retirement home, the probability that exactly 4 of them have a mobility scooter is 24 times greater than the probability that exactly 6 of them have one.

א. What is the probability that a resident who is randomly chosen has a mobility scooter?

ב. 6 of the retirement home's residents are randomly chosen. We know that at least 3 of them have a mobility scooter.

What is the probability that exactly 4 of them have a mobility scooter?

ג. Residents of the retirement home are randomly chosen one after another, until exactly 3 of them have a mobility scooter.

What is the probability that exactly 6 residents will be chosen in this manner?

Part Two — Planar Geometry and Planar Trigonometry

(20 points)

Answer one of the questions 4-5.

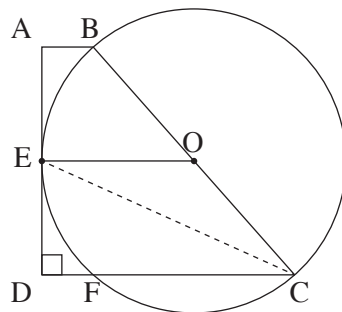
Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. Given: a circle whose center is O.

ABCD is a right trapezoid [טרפז ישר זווית] ($AB \parallel DC$, $\angle ADC = 90^\circ$).

Side AD is tangent to the circle at Point E,
and Points B and C are located on the circle,
such that BC is a diameter.

Side DC intersects the circle at Point F,
as shown in the figure.



א. Prove that $\angle BCD = 2\angle DEF$.

ב. Prove that $\triangle ABE \cong \triangle DFE$.

ג. Prove that $BC = DF + DC$.

5. ABC is an isosceles triangle [משולש שווה שוקיים] ($AB = BC$).

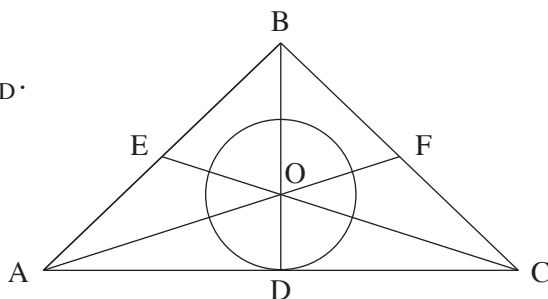
In this triangle, AF, CE and BD are medians [תיכונים] that intersect at Point O (see figure).

א. Prove that $S_{\triangle BOE} = S_{\triangle COD}$.

A circle whose center is O is
tangent to Side AC at Point D.

Given: The area of the circle
is equal to the area of

Triangle AOC.



ב. Calculate the size of the angle ACE.

ג. Express the length of the segment OE in terms of the radius of the circle.

Continued on page 5



**Part Three — Differential and Integral Calculus
of Polynomials, Root Functions, Rational Functions and
Trigonometric Functions (40 points)**

Answer two of the questions 6-8 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x) = \frac{x-5}{\sqrt{x^2-10x+24}}$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
(2) Find the points at which the graph of the function $f(x)$ intersects the axes (if there are such points).
(3) Find the asymptotes of the function $f(x)$ that are perpendicular [מאונכות] to the axes.
(4) Find the intervals on which the function $f(x)$ increases and decreases (if there are such intervals).
(5) Sketch the graph of the function $f(x)$.

Given: the function $g(x)$, for which $g(x) = f(x+5)$.

- ב. (1) Prove that $g(x)$ is an odd function.
(2) Sketch the graph of the function $g(x)$.

ג. Explain why the equality $\int_a^b g(x)dx = \int_{a+5}^{b+5} f(x)dx$ is true for every $1 < a < b$.



7. Given: The function $f(x) = \frac{2 \sin x}{\cos^3 x}$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the points at which the graph of the function $f(x)$ intersects the axes.
- (3) Find the vertical asymptotes of the function $f(x)$.
- (4) Find the intervals on which the function $f(x)$ increases and decreases (if there are such intervals).
- ב. Sketch the graph of the function $f(x)$ on the interval $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.
- ג. Given: $0 < a < \frac{\pi}{2}$.

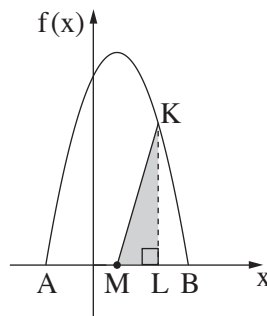
The area of the region that is bounded by the graph of the function $f(x)$, the line $x = a$, and the x -axis is equal to 1.

Find the value of a .

8. The figure on the right depicts the graph of the function $f(x) = -x^2 + 2x + c$ in the range in which it is non-negative [בתחום האי-שליליות].
A and B are the points at which the function $f(x)$ intersects the x -axis.

Given: $x_A = -t$; $x_B = 2t$ ($t > 0$).

- א. Find t and c .



M is the point at which the parabola's axis of symmetry intersects the x -axis.
K is some point on the graph of the function $f(x)$ that is located above the x -axis.

A perpendicular line that intersects the segment AB at Point L was drawn from Point K to the x -axis.

- ב. For what x -coordinates of Point K is the area of Triangle KLM greatest [מקסימלי]?

Find both possible solutions.

You may leave a root in your answer.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל.
אין להעתיק או לפרסם אלא ברשות משרד החינוך